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Scalar field vacuum expectation value induced by gravitational wave background

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ABSTRACT

We show that a *massless* scalar field in a gravitational wave background can develop a non-zero vacuum expectation value. We draw comparisons to the generation of a non-zero vacuum expectation value for a scalar field in the Higgs mechanism and with the dynamical Casimir vacuum. We propose that this vacuum expectation value, generated by a gravitational wave, can be connected with particle production from gravitational waves and may have consequences for the early Universe where scalar fields are thought to play an important role.

1. Introduction

The Brout–Englert–Higgs mechanism [1] is one of the cornerstones of the Standard Model of particle physics. Part of the Higgs mechanism involves a scalar field developing a non-zero vacuum expectation value rather than having a vacuum expectation value of zero. An example of this non-zero vacuum expectation value comes from $\Lambda\Phi^4$ theory with a complex scalar field whose Lagrangian density is

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - \frac{1}{2}m^2|\Phi|^2 - \frac{1}{4}\Lambda|\Phi|^4 \, . \tag{1}$$

The equation of motion from (1) is

$$\partial_\mu \partial^\mu \Phi - m^2\Phi - \Lambda\Phi^3 = 0 \, . \tag{2}$$

If we look for solutions, Φ, which are space and time independent (*i.e.* $\partial_\mu \Phi = 0$) *and* if $m^2 > 0$ then the only solution is $\Phi = 0$. However for a tachyonic mass term (*i.e.* $m^2 < 0$) (2) has a non-zero, constant solution $\Phi_0 = \langle 0|\sqrt{\Phi^*\Phi}|0\rangle = \sqrt{\frac{-m^2}{\Lambda}}$. The vacuum solution is now given by $\Phi = \sqrt{\frac{-m^2}{\Lambda}}e^{i\theta}$ with magnitude $\sqrt{\frac{-m^2}{\Lambda}}$ and a phase $e^{i\theta}$ $(0 \le \theta \le 2\pi)$. Due to the phase of $e^{i\theta}$ there are an infinite number of equivalent vacua labeled by θ. Usually one takes the arbitrary choice of $\theta = 0$ as the vacuum for Φ. This non-zero vacuum expectation value of the scalar field is responsible for giving masses to the $W^\pm$ and Z^0 gauge bosons of the $SU(2) \times U(1)$ Standard Model, while leaving the photon massless.

Aside from the Standard Model, the Higgs mechanism has found application in the theory of superconductors via the Ginzburg–Landau model [2]. In the Ginzburg–Landau model the source of the non-zero order parameter/scalar field vacuum expectation value is due to the interaction between the electrons and the phonons of the background lattice.

Another set of phenomena where a non-trivial vacuum is important are the Casimir effect [3] and dynamical Casimir effect [4]. In the canonical Casimir effect there are two, neutral, conducting plates which are placed a fixed distance apart. This restricts the type of quantum fluctuations that can occur between the plates as compared to outside the plates leading to an attractive force between the plates. In the dynamical Casimir effect the plates are moved with respect to one another and this results in the creation of photons out of the vacuum – a result which has been observed relatively recently [5].

Below we will show that a massless scalar field placed in a gravitational wave background leads to the scalar field developing a non-zero vacuum expectation value. We make a comparison of this gravitationally induced effect with the scalar field vacuum expectation value of spontaneous symmetry breaking as found in the Higgs mechanism and the Ginzburg–Landau model. The comparison to the Ginzburg–Landau model is especially relevant since there the symmetry breaking is driven by the interactions induced

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by the phonons from the background lattice, whereas in the usual Higgs mechanism the symmetry breaking comes from the scalar field's self interaction. As in the Ginzburg–Landau model, here the scalar field's vacuum value is driven by interactions with the gravitational wave background. We also make a comparison between the present gravitationally induced vacuum expectation value and the dynamical Casimir. In the dynamical Casimir effect and the present case there is the possibility of producing *massless* particles from the vacuum. There are earlier works [6] [7] which show that a plane gravitational wave background will not produce particles from the vacuum. We show how this is avoided exactly for the case of *massless* (scalar) fields.

Finally, we connect the results of the present paper with other recent works that propose there is a shift of the pre-existing Higgs vacuum expectation value of the Standard Model either via stationary gravitational fields [8,9] or via a gravitational wave background [10]. There is also very recent work [11] which discusses the consequences of the interaction of a gravitational wave background with a time-dependent vacuum expectation value from a (non-Abelian) gauge field.

2. Scalar field in gravitational wave background

2.1. Approximate gravitational wave background

The equation for a complex scalar field, φ, in a general gravitational background is

$$\frac{1}{\sqrt{-det\left[g_{\mu\nu}\right]}}\left(\partial_\mu g^{\mu\nu}\sqrt{-det\left[g_{\mu\nu}\right]}\partial_\nu\right)\varphi=0. \tag{3}$$

Following [12] we take the gravitational wave to travel in the positive z direction and have the $+$ polarization. For this situation the metric [13] can be written as,

$$\begin{aligned} ds^2 &= -dt^2+dz^2+f(t-z)^2dx^2+g(t-z)^2dy^2 \\ &= dudv+f(u)^2dx^2+g(u)^2dy^2, \end{aligned} \tag{4}$$

where in the last step we have switched to light front coordinates $u=z-t$ and $v=z+t$ with metric components $g_{uv}=g_{vu}=\frac{1}{2}$ and $g_{xx}=f(u)^2$ and $g_{yy}=g(u)^2$. The metric functions $f(u)$ and $g(u)$ will be taken to be oscillatory functions of u. The determinant term in (3) is $\sqrt{-det\left[g_{\mu\nu}\right]}=\frac{fg}{2}$. Substituting the light front version of the metric into equation (3) gives

$$\left(4f^2g^2\partial_u\partial_v+2fg\partial_u(fg)\partial_v+g^2\partial_x^2+f^2\partial_y^2\right)\varphi=0. \tag{5}$$

We take the metric ansatz functions of the form $f(u)=1+\varepsilon(u)$, and $g(u)=1-\varepsilon(u)$ and substitute these into equation (5). This form for $f(u)$ and $g(u)$ describes a wave propagating in the z direction so that x and y directions should be physically identical. Thus we require of the solution that $\left(\partial_y^2-\partial_x^2\right)\varphi=0$. Using this equation (5) becomes,

$$\begin{aligned} &\left[4\left(1-2\varepsilon^2+\varepsilon^4\right)\partial_u\partial_v-4\left(1-\varepsilon^2\right)\varepsilon\left(\partial_u\varepsilon\right)\partial_v \right. \\ &\left. +(1+\varepsilon^2)(\partial_x^2+\partial_y^2)\right]\varphi=0. \end{aligned} \tag{6}$$

Finally we consider a sinusoidal, plane gravitational wave by taking $\varepsilon(u)=h_+e^{iKu}$, where h_+ is a dimensionless amplitude and K is a wave number. The metric must be real so it is understood that the metric components are obtained by taking the real part of the ansatz functions so that $f(u),g(u)=1\pm h_+e^{iKu}\to 1\pm h_+\cos(Ku)$. This real form still satisfies the linearized general relativistic field equations to which $f(u),g(u)$ are solutions. Substituting this choice of $\varepsilon(u)$ into equation (6) gives

$$\left(4F(u)\partial_u\partial_v-4iKG(u)\,\partial_v+H(u)(\partial_x^2+\partial_y^2)\right)\varphi=0, \tag{7}$$

where $F(u)\equiv\left(1-2h_+^2e^{2iKu}+h_+^4e^{4iKu}\right)$, $G(u)\equiv\left(h_+^2e^{2iKu}-h_+^4e^{4iKu}\right)$, and $H(u)=\left(1+h_+^2e^{2iKu}\right)$. We separate equation (7) using $\varphi=X(x)Y(y)U(u)V(v)$. The eigenvalue equations and associated solutions for $X(x)$ and $Y(y)$ are

$$\partial_x^2X=-p^2X\to X(x)=e^{ipx}\quad,\quad \partial_y^2Y=-p^2Y\to Y(y)=e^{ipy}. \tag{8}$$

The function $X(x)$ and $Y(y)$ are simply free waves as is to be expected since the gravitational wave is traveling in the $u=z-t$ direction, and p is the common momentum in the x,y directions. The common momentum in the x and y directions comes from the assumed symmetry in these transverse directions, and it also realizes the condition $\left(\partial_y^2-\partial_x^2\right)\varphi=0$ which we took above. Using (8) we find that (7) becomes

$$F(u)\frac{\partial_uU}{U}\frac{\partial_vV}{V}-iKG(u)\frac{\partial_vV}{V}-H(u)\frac{p^2}{2}=0. \tag{9}$$

Since the light front coordinate v is orthogonal to u and since the gravitational wave only depends on u one expects that the eigenfunction $V(v)$ also is solved by a free, plane wave, as was the case for $X(x)$ and $Y(y)$. This is the case and we find

$$-i\partial_vV=p_vV\to V(v)=e^{ip_vv}. \tag{10}$$

Substituting equation (10) into equation (9) and defining $\lambda\equiv\frac{p^2}{2p_v}$ yields

$$i\frac{\partial_uU(u)}{U(u)}=\lambda\frac{H(u)}{F(u)}-K\frac{G(u)}{F(u)}. \tag{11}$$

The term $i\frac{\partial_uU(u)}{U(u)}$ in (11) represents the kinetic energy of the scalar field; the term $\lambda\frac{H(u)}{F(u)}$ represents the interaction of the scalar field, via λ, with the gravitational background, via $\frac{H(u)}{F(u)}$; the term $K\frac{G(u)}{F(u)}$ represents a pure gravitational potential term. Equation (11) can be integrated to give,

$$U(u)=Ae^{\frac{\lambda}{K}}e^{\overline{\frac{-\lambda}{K\left(1-h_+^2e^{2iKu}\right)}}}\left(1-h_+^2e^{2iKu}\right)^{\frac{1}{2}\left(\frac{\lambda}{K}-1\right)}e^{-i\lambda u}, \tag{12}$$

where $Ae^{\frac{\lambda}{K}}$ is constant. The factor $e^{\frac{\lambda}{K}}$ was chosen to ensure that the eigenfunction for the u direction becomes a free plane wave, $e^{-i\lambda u}$, as $h_+\to 0$ (*i.e.* as the gravitational wave is turned off). Collecting together all the solutions in x,y,v and u directions gives the solution of the scalar field in the gravitational background,

$$\begin{aligned} \varphi &= Ae^{\frac{\lambda}{K}}e^{-\frac{\lambda}{K\left(1-h_+^2e^{2iKu}\right)}}\left(1-h_+^2e^{2iKu}\right)^{\frac{1}{2}\left(\frac{\lambda}{K}-1\right)} \\ &\quad\times e^{-i\lambda u}e^{ip_vv}e^{ipx}e^{ipy}+B. \end{aligned} \tag{13}$$

A is a normalization constant and we have added a constant B which is allowed by the shift symmetry of solutions to (5). Below we choose $B=-A$. This choice of B ensures that if one turns off the gravitational background ($h_+\to 0$) and also takes the field momenta to zero ($\lambda,p_v,p\to 0$) then $\varphi\to 0$. This solution for the scalar field given in (13) is very similar to the form of the solution found in [14] for the *static* electric field evaluated in light front coordinates. Here we have a massless scalar field in a gravitational wave background.