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Anyon black holes

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ABSTRACT

We propose a correspondence between an Anyon Van der Waals fluid and a $(2+1)$ dimensional AdS black hole. Anyons are particles with intermediate statistics that interpolates between a Fermi–Dirac statistics and a Bose–Einstein one. A parameter α $(0 < \alpha < 1)$ characterizes this intermediate statistics of Anyons. The equation of state for the Anyon Van der Waals fluid shows that it has a quasi Fermi–Dirac statistics for $\alpha > \alpha_c$, but a quasi Bose–Einstein statistics for $\alpha < \alpha_c$. By defining a general form of the metric for the $(2+1)$ dimensional AdS black hole and considering the temperature of the black hole to be equal with that of the Anyon Van der Waals fluid, we construct the exact form of the metric for a $(2+1)$ dimensional AdS black hole. The thermodynamic properties of this black hole is consistent with those of the Anyon Van der Waals fluid. For $\alpha < \alpha_c$, the solution exhibits a quasi Bose–Einstein statistics. For $\alpha > \alpha_c$ and a range of values of the cosmological constant, there is, however, no event horizon so there is no black hole solution. Thus, for these values of cosmological constants, the AdS Anyon Van der Waals black holes have only quasi Bose–Einstein statistics.

1. Introduction

The physics of the black holes has always been one of the most interesting research areas since its appearance as a research field [1–7]. It is known that there is an analogy between an AdS black hole and the Van der Waals fluid [8]. The correspondence between the two is important because the thermodynamic behavior of black holes can be explained by that of the fluid; the conventional thermodynamic phase space (including temperature, entropy, and volume) can also be defined for an AdS black hole. This analogy will be more complete in an extended phase space. The past few years has witnessed an interest in the study of the cosmological constant (Λ) as a thermodynamic parameter in the first law of thermodynamics [9,8,10–13]. Although this assumption seems awkward, there are good reasons why Λ should be considered in the first law of thermodynamics. First, including the cosmological constant Λ in the first law of thermodynamics will make it consistent with Smarr relation [10] and the variation of Λ will satisfy the Smarr relation. Second, there exist theories that show physical constants such as c, G, h and Λ are not really constant and have variation with respect to the energy scale of the universe.

Once we introduce the cosmological constant as a thermodynamic parameter, we can define its conjugate variable. Since Λ is proportional to the thermodynamic pressure (for d dimensional space–time $P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2}$ using geometric units $G_N = \hbar = c = k_B = 1$), its conjugate must have volume dimension. This definition will give rise to an additional term, $P\delta V$, in the first law of thermodynamics and the mass of the black hole will be defined in terms of its enthalpy. In this extended phase space, one can write the equation of state for the AdS black hole and compare it with the equation of state for the Van der Waals fluid that reads as follows:

$$T = (P + \frac{a}{v^2})(v - b), \tag{1}$$

where, P is the thermodynamic pressure, v is the specific volume of the fluid $v = \frac{V}{N}$, and N is the degree of freedom (V is the conjugate volume for P). In Ref. [14], the authors derived an exact form for the metric of an AdS black hole which has the same thermodynamics as the Van der Waals fluid. In this work, we construct an Anyon Van der Waals fluid whose thermodynamics is exactly consistent with that of an AdS Anyon Van der Waals black hole.

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What they have in common is that the particles are considered to be either fermions or bosons which obey the Fermi–Dirac or the Bose–Einstein statistics, respectively. For a fluid of the latter type, there can be a Bose–Einstein condensation (For fermions there is no condensation because of Pauli's exclusion principle). But in the $(2+1)$ dimensional space time, we may have an intermediate statistics [15–19]. The particles that have this intermediate statistics are called Anyons. It is notable that fermions and bosons are the two limits of the Anyons. Thus, we can use a real number α $(0<\alpha<1)$ to parameterize the intermediate statistics of the Anyons with $\alpha=0$ corresponding to bosons (the particles that can have the Bose–Einstein condensation), $\alpha=1$ corresponding to fermions (the particles which obey the Pauli exclusion principle), and $0<\alpha<1$ parameterizing the intermediate statistics of the Anyons. Here, we are going to construct a metric for a $(2+1)$ dimensional black hole with statistics consistent with the intermediate statistics of the Anyon fluid. The results show that the Anyon Van der Waals fluid has a quasi Fermi–Dirac statistics for $\alpha_c<\alpha<1$ and that the AdS Anyon Van der Waals fluid has a quasi Bose–Einstein statistics for $0<\alpha<\alpha_c$. In the former case, however, there will be no black hole solution. Thus, for $\alpha_c<\alpha<1$, it is not possible to describe an AdS Anyon Van der Waals black hole by means of an Anyon Van der Waals fluid. The interesting consequence of our work is that AdS Anyon Van der Waals black holes can be expressed only for $0<\alpha<\alpha_c$ and that they have a quasi Bose–Einstein statistics.

The paper is organized as follows:

In Sec. 2, we present a review of the AdS Van der Waals black hole and the equations for both its energy density and pressure. In Sec. 3, the equation of state for the Anyons is introduced. In Sec. 4, the exact form of the metric that is consistent with the AdS Anyon Van der Waals fluid is obtained, the equations of the energy density and the pressure of the black hole are derived, and the behavior of the energy density and the pressure are analyzed. It is interesting that there are black hole solutions that only correspond to the semi Bose Einstein statistics. We present the results and conclusions in Sec. 5.

2. Van der Waals black hole

In [14], the authors constructed a metric for a $3+1$ dimensional AdS black hole that has a similar thermodynamic behavior to that of the Van der Waals fluid and checked the validity of energy conditions for this black hole. This metric construction is based on the AdS Black hole similarity to Van der Waals fluid together with the assumption that the cosmological constant is a thermodynamic variable. In this extended phase space, the relation $P=-\dfrac{\Lambda}{8\pi}=\dfrac{3}{8\pi l^2}$ hold between the thermodynamic pressure and the cosmological constant Λ. By assuming this equation to be true, we should identify the conjugate variable for the pressure proportional to Λ; obviously, the natural choice is volume. So, the equation for the mass of the black hole should be modified from $\delta M=T\delta S$ to:

$$\delta M=T\delta S+V\delta P+\ldots. \tag{2}$$

This is why, in an extended phase space for the AdS black hole, the mass of the black hole is related to its enthalpy [20]. Using Eq. (2), one can see that the thermodynamic volume V can be obtained from:

$$V=\left(\frac{\partial M}{\partial P}\right)_{s,\ldots}. \tag{3}$$

Now if we have the metric of the black hole, we can write the equation of state for the black hole as $P=P(V,T)$ and compare it with that for the fluid. For simplicity, one can assume the metric to be:

$$ds^2=-fdt^2+\frac{dr^2}{f}+r^2d\Omega^2 \tag{4}$$

$$f=\frac{r^2}{l^2}-\frac{2M}{r}-h(r,P), \tag{5}$$

$$M=\frac{4}{3}\pi r_+^3P-\frac{r_+}{2}h(r_+,P) \tag{6}$$

where, M is the mass of the black hole and $h(r,P)$ should be determined accordingly. We assume this metric to be a solution for the Einstein field equation $G_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi T_{\mu\nu}$. The energy momentum tensor is defined in an orthonormal basis by $T^{\mu\nu}=\rho e_0^\mu e_0^\nu+\sum_i p_i e_i^\mu e_i^\mu$, where ρ is the black hole energy density and p is its pressure. So, the pressure and the energy density of the black hole can be calculated by using the metric in Eq. (4):

$$\rho=-p_1=\frac{1-f-rf'}{8\pi r^2}+P \tag{7}$$

$$p_2=p_3=\frac{rf''+2f'}{16\pi r}-P, \tag{8}$$

with the prime denoting the derivative with respect to r.

One should define the function f such that the equation of state for the black hole is consistent with that of the Van der Waals fluid. The specific volume and temperature of the black hole are defined as functions of the black hole horizon and the thermodynamic pressure,

$$v=\frac{k}{4\pi r_+^2}\left[\frac{4}{3}\pi r_+^3-\frac{r_+}{2}\frac{\partial h(r_+,P)}{\partial P}\right] \tag{9}$$

$$T=\frac{f'}{4\pi}=2r_+P-\frac{h(r_+,P)}{4\pi}-\frac{1}{4\pi}\frac{\partial h(r_+,P)}{\partial r_+}, \tag{10}$$

where, for a d space time dimension, $k=\frac{4(d-1)}{d-2}$, $v=k\frac{V}{N}$ and N is proportional to the horizon area as $N=\frac{A}{L_{pl}^2}$ with $A=4\pi r^2$. Since we expect the equation of state for the AdS black hole to be consistent with that of the Van der Waals fluid, we should compare the equation of state obtained from Eqs. (9) and (10) with that of the Van der Waals fluid. The direct relation between the specific volume and pressure of Van der Waals fluid and the temperature of the black hole will be obtained by combining Eqs. (9), (10), and (1):

$$2r_+P-\frac{h}{4r_+\pi}-\frac{h'}{4\pi}=\left(P+\frac{a}{v^2}\right)(v-b). \tag{11}$$

Where prime denotes the derivative with respect to r_+. By setting $h(r,P)=A(r)-PB(r)$, one can find an solution for Eq. (11). This leads to an equation in the form of $F_1(r)P+F_2(r)=0$ in which F_1 and F_2 are the functions of A and B and their derivatives. By setting $F_1(r)=0$ and $F_2(r)=0$ separately, one can derive the solution for $h(r,P)$; hence, we will have the solutions for the energy density and pressure of the black hole.

In this work, we are going to construct new types of black holes whose statistics completely matches that of the Anyon Van der Waals fluid. Anyons are $(2+1)$ dimensional particles with a statistics that interpolates between Bose–Einstein and Fermi–Dirac statistics. As mentioned, the parameter α is used to identify these particles so that we expect the Anyons to obey the Pauli exclusion principle for $\alpha_c<\alpha<1$ and to have a Bose–Einstein condensation for $0<\alpha<\alpha_c$.