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On the supersymmetrization of Galileon theories in four dimensions ☆

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ABSTRACT

We use on-shell amplitude techniques to study the possible $\mathcal{N}=1$ supersymmetrizations of Galileon theories in $3+1$ dimensions, both in the limit of decoupling from DBI and without. Our results are that (1) the quartic Galileon has a supersymmetrization compatible with Galileon shift symmetry ($\phi \to \phi + c + b_\mu x^\mu$) for the scalar sector and a constant shift symmetry ($\psi \to \psi + \xi$) for the fermion sector, and it is unique at least at 6th order in fields, but possibly not beyond; (2) the enhanced "special Galileon" symmetry is incompatible with supersymmetry; (3) there exists a quintic Galileon with a complex scalar preserving Galileon shift symmetry; (4) one cannot supersymmetrize the cubic and quintic Galileon while preserving the Galileon shift symmetry for the complex scalar; and (5) for the quartic and quintic Galileon, we present evidence for a supersymmetrization in which the real Galileon scalar is partnered with an R-axion to form a complex scalar which only has an ordinary shift symmetry.

1. Introduction and results

Galileon theories are scalar effective field theories (EFTs) with higher derivative self-interactions of the form

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 + \sum_{n=3}^{D+1} g_n (\partial\phi)^2 (\partial\partial\phi)^{n-2}\,, \tag{1}$$

where D is the spacetime dimension. The couplings g_n are generally independent. The characteristic feature of these models is that despite the higher derivatives, the equations of motion are only second order. As a consequence, the Galileons have a well-defined classical field theory limit, free from Ostrogradski ghosts. This feature is strongly atypical among EFTs and make Galileons attractive for model building in cosmology and beyond. The cubic Galileon originally arose in the Dvali–Gabadadze–Porrati (DGP) model [1], but Galileons appear in other contexts too, for example in modifications of gravity [2–4]. Perhaps most significantly, Galileons emerge as subleading terms on effective actions on branes [5]. Here we focus on flat branes in Minkowski space, although other embeddings are also of interest [5–7].

A flat 3-brane placed in a $4+1$-dimensional Minkowski bulk will induce a spontaneous breaking of spacetime symmetry: $\mathrm{ISO}(4,1) \to \mathrm{ISO}(3,1)$. A massless Goldstone mode ϕ must appear in the spectrum of the $3+1$-dimensional world-volume EFT which is physically identified with fluctuations of the brane into the extra dimension. The full $\mathrm{ISO}(4,1)$ symmetry remains a symmetry of the action, and so at leading and next-to-leading order in the derivative expansion the effective action takes the form [5]

$$S = \int d^4x \sqrt{-G}\left[\Lambda_2^4 + \Lambda_3^3 K[G] + \Lambda_4^2 R[G] + \Lambda_5 \mathcal{K}_{\mathrm{GHY}}[G]\right], \tag{2}$$

where G is the pullback of the bulk metric onto the 3-brane world-volume. The leading term with coupling Λ_2^4 (the brane tension) gives the Dirac–Born–Infeld (DBI) action, while the remaining terms are built from the extrinsic curvature, intrinsic curvature and Gibbons–Hawking–York (GHY) boundary terms. The Λ_i are in general arbitrary mass scales. The resulting action is the *DBI-Galileon* [5]: its leading part is DBI and the subleading terms are cubic, quartic, and quintic in ϕ.[1] The non-DBI interaction terms in

[1] The boundary terms $K[G]$ and $\mathcal{K}_{\mathrm{GHY}}[G]$ are only available when the brane is considered an end-of-the-world brane and they are responsible for the odd-powered ϕ-interactions.

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(2) are the 4+1-dimensional Lovelock invariants that give the characteristic second-order equations of motion.

The *Galileon models* (1) correspond to a decoupling limit in which the 3-brane tension $\Lambda_2^4 \to \infty$, but the ratios

$$g_3 = \frac{\Lambda_3^3}{\Lambda_2^6}, \quad g_4 = \frac{\Lambda_4^2}{\Lambda_2^8}, \quad g_5 = \frac{\Lambda_5}{\Lambda_2^{10}}, \tag{3}$$

are held fixed. The only part of DBI that survives is the canonical kinetic term for ϕ.[2]

The Galileons and the DBI-Galileons both enjoy a non-trivial extended shift symmetry of the form

$$\phi \to \phi + c + b_\mu x^\mu + \dots, \tag{4}$$

where c is a constant, b_μ is a constant vector, and x^μ is the spacetime coordinate. The ellipses stand for possible field-dependent terms, which will not play a role for us here. These symmetries arise from the spontaneously broken symmetry generators [6]: the constant shift from the broken bulk translation and the x^μ-shift from the broken Lorentz rotations.

The quartic Galileon ($g_3 = g_5 = 0$) is sometimes called the *special Galileon* [9,10] because it has a further enhanced shift symmetry

$$\phi \to \phi + s_{\mu\nu} x^\mu x^\nu + \dots, \tag{5}$$

where the constant tensor $s_{\mu\nu}$ is symmetric and traceless [10]. This is an accidental symmetry that occurs only in the decoupling limit from DBI.

In this paper, we address the question of supersymmetrization of Galileon theories in 3+1 dimensions, both in the context of DBI-Galileons and the decoupled Galileons. Note that the fermionic superpartners will not have an extended shift symmetry, but only a shift symmetry of the form $\psi \to \psi + \xi$ with ξ a constant spinor.[3]

Based on the brane construction, one expects that the quartic DBI-Galileon can be supersymmetrized, in particular, there should exist an $\mathcal{N} = 4$ supersymmetrization corresponding to the effective action for a D3-brane in 9+1-dimensional Minkowski space. It is less obvious that supersymmetry would survive the decoupling limit or if the cubic or quintic (DBI-)Galileons can be supersymmetrized. An explicit $\mathcal{N} = 1$ superfield construction of the quartic Galileon was presented in [12]. We will construct an $\mathcal{N} = 1$ quartic Galileon and comment on its uniqueness. In the literature, a supersymmetrization of the cubic Galileon was proposed, but it suffered from ghosts [13]. By a field redefinition, any cubic Galileon is equivalent to the quartic and quintic Galileon with related couplings, so we address supersymmetrization of the cubic Galileon via the quintic.[4]

Before describing our approach, we comment briefly on the super-algebra.

1.1. Symmetry algebra

The Poincare algebra can be extended [10,17] with the translation generator C ($\delta_C \phi = 1$), the Galileon shift generator B_μ ($\delta_B \phi = x^\mu$), and the symmetric traceless generator $S_{\mu\nu}$ of the special Galileon transformations (5).

Being agnostic about the origin of a Galileon extension of the super-Poincare algebra, at the minimum we might demand the closure of the extended super-translation sub-algebra with generators P_μ, Q, $\bar{Q}$, C, and B_μ (plus $S_{\mu\nu}$ for the special Galileon), as well as a second set of fermionic generators S and $\bar{S}$ associated with spontaneously broken supersymmetry. The latter are required by the algebra. Among the new commutator relations, we must have (schematically)

$$\begin{aligned} &[P_\mu, B_\nu] \sim \eta_{\mu\nu} C, \quad [P_\rho, S_{\mu\nu}] \sim \eta_{\mu\rho} B_\nu + \eta_{\nu\rho} B_\mu, \\ &[B_\mu, Q] \sim \sigma_\mu (\bar{Q} + \bar{S}), \quad [S_{\mu\nu}, Q] = 0. \end{aligned} \tag{6}$$

The last vanishing commutator follows from the fact that $[S_{\mu\nu}, Q]$ must be a linear-combination of fermionic generators, but there are no tensor structures available that can make it symmetric and traceless. Now, consider the Jacobi identity

$$[S_{\mu\nu}, \{Q, \bar{Q}\}] = \{[S_{\mu\nu}, Q], \bar{Q}\} + \{Q, [S_{\mu\nu}, \bar{Q}]\}. \tag{7}$$

The RHS vanishes, but using $\{Q, \bar{Q}\} \sim P$ the LHS gives a non-vanishing linear combination of B_μ-generators. Therefore the algebra does not close consistently. This indicates that there is no supersymmetrization of the special Galileon that also preserves the enhanced symmetry (5). Replacing $S_{\mu\nu}$ by B_μ in the Jacobi identity (7) gives C on the LHS. The RHS can match this if $\{Q, S\} \sim C$. There does not appear to be any inconsistency extending the super-translation algebra with the Galileon generators C and B_μ. Indeed, such an algebra follows from the scenario of bulk supersymmetry spontaneously broken to $\mathcal{N} = 1$ on the 3-brane.[5]

These algebraic arguments constrain the form of the symmetry as realized on the classical fields and are suggestive but formally problematic when extended to the quantum theory. In general, spontaneously broken symmetries do not possess well-defined Noether charges as operators on a Hilbert space.[6] As demonstrated in [21], the infinite volume improper integral of the Noether charge density operators of spontaneously broken symmetries do not converge in the weak operator topology. Furthermore, the second S-type supersymmetry is necessarily spontaneously broken and satisfies a current algebra with tensor central charges which cannot be integrated to a consistent charge algebra in infinite volume. (See [22] for a related discussion.) It is difficult to draw convincing conclusions from an algebra which formally does not exist.

Nonetheless we will find that the properties suggested by the algebraic arguments do indeed hold as properties of the scattering amplitudes and can be argued for in a mathematically satisfactory way. In the following, we outline the strategy of using on-shell amplitudes to assess the existence of effective field theories with linearly and non-linearly realized supersymmetry. We then apply these methods to prove each of claims in the Abstract.

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[2] When decoupled from DBI, Galileons violate the null-energy condition and for that reason they have received attention as models for cosmological bounces. However, without the leading DBI terms, the Galileon theories cannot arise as the low-energy limit of a UV complete theory [8].

[3] Fermions with soft behavior $\sigma = 2$ may occur in a fermionic theory whose leading interaction is quartic starting at couplings of mass dimension −8 [11]; this is subleading to Galileons and therefore not relevant here.

[4] In the literature, one also finds studies of conformal Galileon theories with supersymmetry, see for example [14–16]. Conformal Galileons can be thought of as the subleading terms of the effective action of branes in AdS space and they have rather different properties than the Galileons studied here. For example, the amplitudes have different soft behavior.

[5] The decoupling limit (3) induces an İnönü–Wigner contraction of the original ISO(4, 1) symmetry algebra in the direction transverse to the 3-brane. The resulting algebra $\mathfrak{Gal}(4,1)$ is a cousin of the familiar Galilean algebra of non-relativistic mechanics. In the decoupling limit (3), the extended shift symmetry (4) arises from the non-linear realization of the coset $\mathfrak{Gal}(4,1)/\mathrm{ISO}(3,1)$. The recent work [18] extends this construction to include the supercharges. An earlier version of the algebra is in [19].

[6] The algebra constructed in [18] is of the former kind. In this case even the classical Poisson algebra will differ from the algebra realized on the fields by the appearance of central terms [20].