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Fluid dynamics of out of equilibrium boost invariant plasmas

Jean-Paul Blaizot [a], Li Yan [b,*]

[a] Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France
[b] Department of Physics, McGill University, 3600 rue University, Montréal, QC H3A 2T8, Canada

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ABSTRACT

By solving a simple kinetic equation, in the relaxation time approximation, and for a particular set of moments of the distribution function, we establish a set of equations which, on the one hand, capture exactly the dynamics of the kinetic equation, and, on the other hand, coincide with the hierarchy of equations of viscous hydrodynamics, to arbitrary order in the viscous corrections. This correspondence sheds light on the underlying mechanism responsible for the apparent success of hydrodynamics in regimes that are far from local equilibrium.

1. Introduction

The observation that the evolution of the quark–gluon plasma produced in ultra-relativistic heavy ion collisions is well described by viscous hydrodynamic equations raises a number of interesting questions that are very much debated presently [1]. Traditional understanding of hydrodynamics would imply that the system has reached local equilibrium, and the small viscosity extracted from the analysis of the data is suggestive of short mean free paths. However, works on strongly coupled plasmas, using in particular holography techniques, indicate that viscous hydrodynamics works even when large anisotropies, that signal departure from local equilibrium, are still present [2]. At the same time, there is evidence that hydrodynamics is capable of describing small colliding systems, for which no clear separation a priori exists between microscopic and macroscopic scales (see e.g. the recent discussion in [1,3] and references therein).

Recently, it has been argued that part of the success of hydrodynamics could be due to the existence of a stable attractor, to which the solution of the dynamical equations quickly converge before eventually reaching the viscous hydrodynamic regime [4]. This suggestion has triggered many studies, some of which involve sophisticated mathematical developments [5–9]. In this paper, we would like to offer an alternative perspective on the issue, based on the simple, and physically motivated observation, that the main features of the dynamics of expanding plasmas are determined by the competition between the expansion itself, which is dictated by the external conditions of the collisions, and the collisions among the plasma constituents which generically tend to isotropize the particle momentum distribution functions. These two competing effects give rise to two independent fixed points of a suitably defined dynamical quantity. Many recent results find a natural interpretation in the interplay between these two fixed points.

As in many works on this issue, we focus on the paradigmatic example of the Bjorken flow [10], and consider an expanding system of massless particles characterized by a distribution function f whose time evolution is given by a kinetic equation. Symmetry allows us to reconstruct the full space–time history of the system from the knowledge of what happens in a slice centered around the plane $z = 0$ where the collision takes place. The distribution function in that slice depends solely on the momentum of the particle and the (proper) time τ, i.e., $f = f(\mathbf{p}, \tau)$. Using a relaxation time approximation for the collision kernel, we can then write the following simple kinetic equation [11]

$$\left[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right] f(\mathbf{p}, \tau) = -\frac{f(\mathbf{p}, \tau) - f_{\rm eq}(p/T)}{\tau_R}. \tag{1}$$

Here $f_{\rm eq}(p/T)$ is a function that depends only on $p = |\mathbf{p}|$ and an effective temperature $T(\tau)$ which is determined by requiring that the energy density calculated with $f_{\rm eq}(p/T)$ and $f(\mathbf{p}, \tau)$ takes the same value, $\varepsilon \propto T^4$, at all times. The kinetic equation (1) makes transparent the competition alluded to above, between the expansion and the collisions. In the absence of the collision term, the expansion, controlled by the term $-p_z/\tau$ in the left hand side, leads to a flattened distribution, $f(\mathbf{p}, \tau) \to f_0(p_z\tau, \mathbf{p}_\perp)$, where f_0 is the initial distribution and $\mathbf{p}_\perp$ is the component of the momentum orthogonal to the z-axis. On the other hand, the collision term in the right hand side drives the distribution towards isotropy, at a rate controlled by the relaxation time τ_R.

* Corresponding author.
E-mail address: li.yan@physics.mcgill.ca (L. Yan).

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2. Kinetics in terms of $\mathcal{L}$-moments

Although Eq. (1) can be easily solved numerically, more insight can be gained by using an alternative, albeit approximate, approach that eliminates from the description as much of irrelevant information as possible. Thus, in this paper, instead of considering the full distribution $f(\mathbf{p},\tau)$, we focus on some of its moments, introduced in Ref. [12]:

$$\mathcal{L}_n = \int \frac{d^3\mathbf{p}}{(2\pi)^3 p^0} |\mathbf{p}|^2 P_{2n}(p_z/|\mathbf{p}|) f(\mathbf{p},\tau), \tag{2}$$

where P_{2n} is a Legendre polynomial of order $2n$. The moments $\mathcal{L}_n$ with $n \geq 1$ describe the momentum anisotropy of the system. In particular $\mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T$ reflects the asymmetry between longitudinal (P_L) and transverse (P_T) pressures. The moment $\mathcal{L}_0$ coincides with the energy density, $\mathcal{L}_0 = \varepsilon = P_L + 2P_T$. Observe that the momentum weight of the integration in Eq. (2) is always $|\mathbf{p}|^2$, instead of being an increasing power of $|\mathbf{p}|$ as is the case in more standard approaches (see e.g. [13]). Thus, the $\mathcal{L}_n$'s contain little information on the radial shape of the momentum distribution, preventing us for instance to reconstruct from them the full distribution. However, this radial shape plays a marginal role in the isotropization of the momentum distribution, which is our main concern here. Note that all the $\mathcal{L}_n$ have the same dimension.

By using the recursion relations among the Legendre polynomials, we can recast Eq. (1) into the following (infinite) set of coupled equations

$$\begin{aligned} \frac{\partial \mathcal{L}_n}{\partial \tau} &= -\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R} \quad (n \geq 1) \\ \frac{\partial \mathcal{L}_0}{\partial \tau} &= -\frac{1}{\tau}\left[a_0 \mathcal{L}_0 + c_0 \mathcal{L}_1\right], \end{aligned} \tag{3}$$

where the coefficients a_n, b_n, c_n are pure numbers

$$\begin{aligned} a_n &= \frac{2(14n^2+7n-2)}{(4n-1)(4n+3)}, \quad b_n = \frac{(2n-1)2n(2n+2)}{(4n-1)(4n+1)}, \\ c_n &= \frac{(1-2n)(2n+1)(2n+2)}{(4n+1)(4n+3)}, \end{aligned} \tag{4}$$

entirely determined by the free streaming part of the kinetic equation. Note that the collision term does not affect directly the energy density, but only the moments with $n \geq 1$. In fact, if one ignores the expansion, i.e., set $a_n = b_n = c_n = 0$, the moments evolve according to

$$\mathcal{L}_0(\tau) = \mathcal{L}_0(0), \quad \mathcal{L}_n(\tau) = \mathcal{L}_n(0)\, \mathrm{e}^{-\tau/\tau_R}. \tag{5}$$

This solution illustrates the role of the collisions in erasing the anisotropy of the momentum distribution as the system approaches equilibrium. Of course, the expansion prevents the system to ever reach this trivial equilibrium fixed point: instead, the system goes into an hydrodynamical regime, as we shall discuss later.

The system of Eqs. (3) lends itself to simple truncations. Thus by ignoring all moments of order higher than n, one obtains a finite set of $n+1$ equations that can be easily solved. The accuracy of such a procedure can be judged from Fig. 1, where the moments obtained from various truncations are compared with those of the numerical solution of Eq. (1) for an initial distribution typical of a heavy ion collision: $f(\tau_0, p_T, p_z) = f_0 \Theta\left(Q_s - \sqrt{\xi^2 p_z^2 + p_T^2}\right)$ with $f_0 = 0.1$, $\xi = 1.5$, corresponding to an initial momentum anisotropy $\mathcal{P}_L/\mathcal{P}_T \approx 0.5$, and $\tau_0 = Q_s^{-1}$ [12]. Already the lowest order truncation at $n = 1$ captures the qualitative behavior of the full solution. Note that the approach to the exact solution is alternating, which offers an estimate of the truncation error. The energy density approaches smoothly the hydrodynamic regime as $\tau \gtrsim \tau_R$, while the non monotonous behavior of the ratio $\mathcal{L}_1/\mathcal{L}_0$ reflects the competition between expansion and collisional effects that we now analyze in more detail, starting with the free streaming regime.

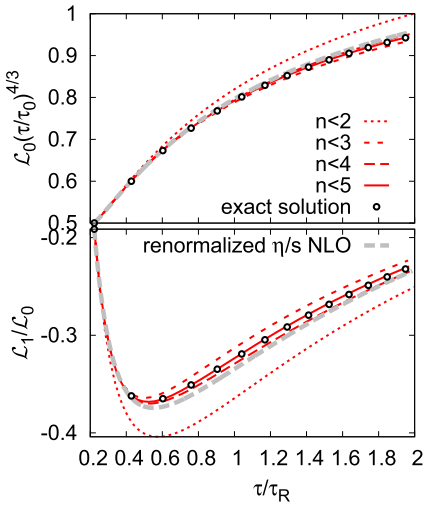

Fig. 1. (Color online) Comparison of the $\mathcal{L}$-moment equations obtained from various truncation of Eqs. (3) (lines), with those of the numerical solution of the kinetic equation (1) (symbols).

3. The free streaming fixed point

The free streaming regime is described by Eq. (3) where one ignores the collision term. It is not hard to see that the resulting equation possesses a stable solution at large time, in which all moments decay as $1/\tau$ and are proportional to each other: $\mathcal{L}_n(\tau) = A_n \mathcal{L}_0(\tau)$, where the dimensionless constants A_n characterize the moments of a distribution that is flat in the p_z direction [12]

$$A_n = P_{2n}(0) = (-1)^n \frac{(2n-1)!!}{(2n)!!}. \tag{6}$$

Note that $A_1 = -1/2$, corresponding to a vanishing longitudinal pressure. As for the factor $1/\tau$, it reflects the conservation of the energy in the increasing comoving volume. Defining

$$g_n(\tau) = \tau\, \partial_\tau \ln \mathcal{L}_n, \tag{7}$$

we get from Eq. (3)

$$g_n(\tau) = -a_n - b_n \frac{\mathcal{L}_{n-1}}{\mathcal{L}_n} - c_n \frac{\mathcal{L}_{n+1}}{\mathcal{L}_n} - (1-\delta_{n0})\frac{\tau}{\tau_R}. \tag{8}$$

The solution above corresponds to a fixed point for the g_n's. Dropping the last term, and using the expression (6) for the ratio of moments, one indeed verifies easily that for all n, $g_n(\tau) = -1$. If the initial ratios of moments are chosen according to Eq. (6), the g_n's remain constant in time (all equal to -1), whereas for arbitrary initial conditions, they will reach the fixed point at late time. Note that the fixed point obtained from a truncation at a finite order differs slightly from -1: for instance, in the simplest truncation at $n < 2$, $g_0 = g_1 = -0.92937$ instead of -1, and $A_1 \approx -0.6$ instead of -0.5.