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General Relativity solutions in modified gravity

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Recent gravitational wave observations of binary black hole mergers and a binary neutron star merger by LIGO and Virgo Collaborations associated with its optical counterpart constrain deviation from General Relativity (GR) both on strong-field regime and cosmological scales with high accuracy, and further strong constraints are expected by near-future observations. Thus, it is important to identify theories of modified gravity that intrinsically possess the same solutions as in GR among a huge number of theories. We clarify the three conditions for theories of modified gravity to allow GR solutions, i.e., solutions with the metric satisfying the Einstein equations in GR and the constant profile of the scalar fields. Our analysis is quite general, as it applies a wide class of single-/multi-field scalar–tensor theories of modified gravity in the presence of matter component, and any spacetime geometry including cosmological background as well as spacetime around black hole and neutron star, for the latter of which these conditions provide a necessary condition for no-hair theorem. The three conditions will be useful for further constraints on modified gravity theories as they classify general theories of modified gravity into three classes, each of which possesses i) unique GR solutions (i.e., no-hair cases), ii) only hairy solutions (except the cases that GR solutions are realized by cancellation between singular coupling functions in the Euler–Lagrange equations), and iii) both GR and hairy solutions, for the last of which one of the two solutions may be selected dynamically.

1. Introduction

Recent measurements of gravitational waves (GWs) from binary black hole (BH) mergers by LIGO and Virgo Collaborations [1,2] clarified that the observed GWs are consistent with the prediction of General Relativity (GR) for binary coalescence waveforms. Moreover, the almost simultaneous detection of GWs from a neutron star (NS) merger [3], and the short gamma-ray burst [4] has significantly constrained a deviation of propagation speed of GWs over cosmological distance from the speed of light down order 10^{-15} [5]. The future measurements of GWs with unprecedented accuracies will make it possible to test modified gravity from completely different aspects.

Various gravitational theories alternative to GR have been proposed to explain inflation and/or late-time acceleration of the Universe [6]. Scalar-tensor theories of gravitation involve the representative frameworks for modification of GR such as Horndeski theory [7] (or generalized Galileon [8–12]), and even today sensible construction of scalar–tensor theories have been extensively investigated [13–21]. The possible deviations from astrophysical and cosmological predictions in GR have been explored as smoking guns of these theories [6,22,23].

The situation changes abruptly by the recent GW observations. The constraint on the propagation speed of GWs severely restricts theories of modified gravity for the late-time accelerated expansion [24–29] and those with the screening mechanism [30–33]. Moreover, the worldwide network of GW interferometer will include KAGRA [34], and further improve these tests of gravity both on strong-field regime and cosmological scales. Within next few years, it is plausible that no deviation from predictions in GR would be detected. If it is the case, GR or modified gravity theories sharing the same background solutions and perturbation dynamics with GR would be observationally preferred.[1]

[1] It should be emphasized that even if GR and modified gravity theories share the same background solution, it is not necessarily true that the perturbation dynamics is also the same in both theories, as firstly addressed in Ref. [35] for specific theories. Nevertheless, our point is that if the observational data agree with the predictions of the perturbations in GR, it would suggest that the background solution is given by a GR solution.

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It is then important to note that no detection of deviation from GR predictions does not immediately exclude modified gravity theories especially in strong-field regime, as many theories could share the same solutions with GR. In GR, there is the no-hair theorem which states that the BH spacetime is solely determined by three conserved quantities or "hairs"; mass, angular momentum, and electric charge [36–38]. In general, scalar–tensor theories may possess BH solutions with nontrivial scalar hair [39–57] which are different from GR BH solutions with the constant profile of the scalar fields. Interestingly, however, there exist some class of modified gravity theories allowing only the BH metric solutions in GR with constant scalar field as the unique solutions [58–67]. This is the extension of no-hair theorems, and implies that these classes evade constraints on deviation of BH spacetime from GR one. Moreover, even in a case where GR and non-GR BH solutions exist simultaneously and the GR BH solution is not the unique solution, if it is the late-time attractor, the theory dynamically selects the GR BH solution and still evades the constraints. Therefore, taking into account the rapidly expanding frontier of the modified gravity theories and the remarkable progress of their constraints from GW observations, it is important to identify which class of the most general scalar–tensor theories could admit GR BH solutions.

In this Letter, we clarify the conditions for the existence of GR solutions in a quite general scalar–tensor theory defined by (1) below, where by "GR solution" we mean a solution with a metric satisfying the Einstein equations in GR and a constant profile of the scalar fields. Our analysis will expand that in Ref. [68] which showed that different gravitational theories share the Kerr solution same as in GR. Ref. [69] constructed the higher-order Ricci polynomial gravity theories that admit the same vacuum static solutions as GR. We will cover modified gravity theories which can be described by any class of single-/multi-field scalar–tensor theories. Our analysis solely exploits the covariant equations of motion without assuming any symmetry and ansatz for the metric and scalar fields, and hence any GR solution is within our subject. Note that "GR solution" here represents not only static or stationary BH solutions such as Schwarzschild, Kerr, and Schwarzschild–de Sitter solutions, but also any solution in GR in astrophysical or cosmological situation with/without the existence of matter. Our analysis will also apply higher dimensional spacetime, in which a caveat is that vacuum GR solutions include not only spherical BHs, but also black objects with nonspherical horizon topology [70,71], and hence the uniqueness of black objects does not hold.

It should be emphasized that our analysis focuses on GR solutions with the constant profile of the scalar fields, and there are several theories that do not fit our analysis, e.g., theories with self-gravitating media such as Lorentz-violating massive gravity [72–77], and theories where the small-scale behavior such as breaking of the Vainshtein screening is sensitive to the asymptotic time-dependence of the scalar fields [78–80]. Correspondingly, there are also several examples of BH solutions with the metric of GR in modified gravity theories that are not captured by the constant scalar field ansatz, e.g., the Schwarzschild–de Sitter BHs in the shift-symmetric Horndeski theories [49] and in the massive gravity theories [81–85], and the Kerr solution in the purely quartic Horndeski theory [57].

2. The model

We consider a wide class of single-/multi-field scalar–tensor theories in D-dimensional spacetime described by the action

$$S=\int d^Dx\sqrt{-g}\big[G_2(\phi^I,X^{JK})+G_4(\phi^I,X^{JK})R + \phi^I_{;\mu_1}C^{\mu_1}_{1I}+\phi^I_{;\mu_1\mu_2}C^{\mu_1\mu_2}_{2I}+\phi^I_{;\mu_1\mu_2\mu_3}C^{\mu_1\mu_2\mu_3}_{3I}+\cdots + L_m(g_{\mu\nu},\psi)\big], \tag{1}$$

where the Greek indices $\mu,\nu,\cdots$ run the D-dimensional spacetime, the capital Latin indices $I,J,\cdots$ label the multiple scalar fields, and semicolons denote the covariant derivative with respect to the metric $g_{\mu\nu}$. In addition to the Ricci curvature R and the matter Lagrangian $L_m(g_{\mu\nu},\psi)$ minimally coupled to gravity, the action involves arbitrary functions: G_2, G_4 are functions of the multiple scalar fields ϕ^I and the kinetic terms $X^{IJ}\equiv -g^{\mu\nu}\phi^I_{;\mu}\phi^J_{;\nu}/2$, and $C^{\mu_1}_{1I}, C^{\mu_1\mu_2}_{2I}, C^{\mu_1\mu_2\mu_3}_{3I},\cdots$ are functions of $(g_{\alpha\beta}, g_{\alpha\beta,\gamma}, g_{\alpha\beta,\gamma\delta},\cdots;\phi^I,\phi^I_{;\alpha},\phi^I_{;\alpha\beta},\cdots;\epsilon_{\mu\nu\rho\sigma})$ with $\epsilon_{\mu\nu\rho\sigma}$ being the Levi-Civita tensor. The dots in (1) contain contractions between arbitrary higher-order covariant derivatives of a scalar field and its corresponding C-function, $\phi^I_{;\mu_1\cdots\mu_n}C^{\mu_1\cdots\mu_n}_{nI}$. In order for Eq. (1) to be covariant with respect to $g_{\mu\nu}$, the dependence of $C^{\mu_1}_{1I}, C^{\mu_1\mu_2}_{2I}, C^{\mu_1\mu_2\mu_3}_{3I},\cdots$ on the metric should be through metric itself, curvature tensors associated with it, and their covariant derivatives.

This action is very generic and covers a lot of single-/multi-field models of scalar–tensor theories. Indeed, the term $\phi_{;\mu\nu}C^{\mu\nu}_2$ includes Ostrogradsky ghost-free single-field scalar–tensor theories such as Horndeski [7] (generalized Galileon [8–12]), Gleyzes–Langlois–Piazza–Vernizzi (GLPV) [14,15], and degenerate higher-order scalar–tensor (DHOST) theories [17,20] as a subclass. Specifically, the Horndeski action in the four-dimensional spacetime is described by $C^{\mu\nu}_2=C^{\mu\nu}_{\rm H}$ with

$$C^{\mu\nu}_{\rm H}=G_3g^{\mu\nu}+G_{4X}(g^{\mu\nu}\Box\phi-\phi^{;\mu\nu})+G_5G^{\mu\nu} -\frac{1}{6}G_{5X}[g^{\mu\nu}(\Box\phi)^2-3\Box\phi\phi^{;\mu\nu}+2\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}], \tag{2}$$

and GLPV action is given by $C^{\mu\nu}_2=C^{\mu\nu}_{\rm H}+C^{\mu\nu}_{\rm bH}$ with

$$C^{\mu\nu}_{\rm bH}=F_4\epsilon^{\alpha\beta\mu}{}_{\gamma}\epsilon^{\tilde\alpha\tilde\beta\nu\gamma}\phi_{;\alpha}\phi_{;\tilde\alpha}\phi_{;\beta\tilde\beta} +F_5\epsilon^{\alpha\beta\gamma\mu}\epsilon^{\tilde\alpha\tilde\beta\tilde\gamma\nu}\phi_{;\alpha}\phi_{;\tilde\alpha}\phi_{;\beta\tilde\beta}\phi_{;\gamma\tilde\gamma}, \tag{3}$$

where G_n, F_n are functions of ϕ, $X=-g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}/2$, and $G_{nX}\equiv\partial G_n/\partial X$. Likewise, it is also clear that quadratic- and cubic-order DHOST theories are a subclass and described by the $\phi_{;\mu\nu}C^{\mu\nu}_2$ term. It also includes parity-violating theories with Chern–Simons term or Pontryagin density $\epsilon_{\alpha\beta\gamma\delta}R^{\alpha\beta}{}_{\mu\nu}R^{\gamma\delta\mu\nu}/2$ [86–95], the multi-Galileon theories [96–104], those with complex scalar fields, and even more general higher-order theories involving derivatives higher than second order, which can be free from the Ostrogradsky ghost by imposing a certain set of ghost-free conditions [16,21]. Note that in this paper we will focus only on the conditions for obtaining the GR solutions and actually it does not matter whether the theory (1) contains the Ostrogradsky ghost or not. Hence, the following analysis for (1) to allow GR solutions is powerful and exhausts almost all the known scalar–tensor theories of modified gravity.

3. Conditions for GR solutions

We focus on a solution in GR with a given value of cosmological constant Λ for $\Phi^I\equiv(\phi^I,\phi^I_{;\alpha},\phi^I_{;\alpha\beta},\cdots)=\Phi^I_0$, where $\Phi^I_0\equiv(\phi^I_0,0,0,\cdots)$ and ϕ^I_0 is constant, which satisfies the Einstein equation

$$G^{\mu\nu}=8\pi GT^{\mu\nu}-\Lambda g^{\mu\nu}, \tag{4}$$

where $T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g_{\mu\nu}}$ is the stress energy tensor for the matter component, which is further decomposed into the classical and constant parts $T^{\mu\nu}=T^{\mu\nu}_m-(8\pi G)^{-1}\Lambda_m g^{\mu\nu}$, where the latter denotes the contribution of matter vacuum fluctuations. We elucidate