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On the stability of a superspinar

Ken-ichi Nakao [a,*], Pankaj S. Joshi [b], Jun-Qi Guo [b,c], Prashant Kocherlakota [b], Hideyuki Tagoshi [d], Tomohiro Harada [e], Mandar Patil [f], Andrzej Królak [f]

[a] Department of Mathematics and Physics, Graduate School of Science, Osaka City University, 3-3-138 Sugimoto, Sumiyoshi, Osaka 558-8585, Japan
[b] Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India
[c] School of Physics and Technology, University of Jinan, No. 336, West Road of Nan Xinzhuang, Jinan 250022, Shandong, China
[d] Institute for Cosmic Ray Research (ICRR), University of Tokyo, 5-1-5 Kashiwa-no-ha, Kashiwa, Chiba 277-8582, Japan
[e] Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan
[f] Institute of Mathematics, Polish Academy of Sciences, Śniadeckich 8, 00-656 Warsaw, Poland

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ABSTRACT

The superspinar proposed by Gimon and Hořava is a rapidly rotating compact entity whose exterior is described by the over-spinning Kerr geometry. The compact entity itself is expected to be governed by superstringy effects, and in astrophysical scenarios it can give rise to interesting observable phenomena. Earlier it was suggested that the superspinar may not be stable but we point out here that this does not necessarily follow from earlier studies. We show, by analytically treating the Teukolsky equations by Detwiler's method, that in fact there are infinitely many boundary conditions that make the superspinar stable at least against the linear perturbations of $m = l$ modes, and that the modes will decay in time. Further consideration leads us to the conclusion that it is possible to set the inverse problem to the linear stability issue: since the radial Teukolsky equation for the superspinar has no singular point on the real axis, we obtain regular solutions to the Teukolsky equation for arbitrary discrete frequency spectrum of the quasi-normal modes (no incoming waves) and the boundary conditions at the "surface" of the superspinar are found from obtained solutions. It follows that we need to know more on the physical nature of the superspinar in order to decide on its stability in physical reality.

1. Introduction

The Kerr spacetime is an exact stationary solution of the vacuum Einstein equations and is characterized by two parameters, namely the gravitational mass M and the so-called Kerr parameter a which is the angular momentum divided by M. The solution describes a rotating black hole if $a^2 \leq M^2$, whereas it describes a naked singular spacetime if $a^2 > M^2$, using the geometrized units ($G = c = 1$). The Kerr black hole has been extensively studied in many scenarios which would be stable against linear perturbations. This may suggest the reliability of the weak version of the cosmic censorship hypothesis whose statement is, roughly speaking, the spacetime singularities formed from generic initial conditions are enclosed by event horizons. Also, many black-hole candidates, i.e. objects described by the Kerr solution of $a^2 < M^2$ have been found in our universe.

Gimon and Hořava pointed out an interesting fact that the supersymmetry does not imply the Kerr bound $a^2 cleq M^2$, and hence if a very compact object of $a^2 > M^2$ is found, it may be a signal of superstring theory [1]. They named it the superspinar. The naked singularity will be made harmless by stringy effect. However, before the indication of Gimon and Hořava, a study suggested the instability of the over-spinning Kerr spacetime $a^2 > M^2$ [2]. After the superspinar possibility, few more studies were done on the stability of the over-spinning Kerr geometry by other researchers [3–5], to suggest that the superspinar is unstable under various boundary conditions. The variety of the boundary conditions is maximal in the study by Pani et al., which includes all the previous studies, and they concluded that the over-spinning Kerr geometry and thus the superspinar is unstable. However, it should be noted that in order to conclude so, we must show that the over-spinning Kerr geometry is unstable under all possible boundary conditions, since at present nobody knows the physical nature of the superspinar. From this standpoint, the numerical results obtained by Pani et al. may not necessarily imply the instability of the superspinar.

* Corresponding author.
E-mail address: knakao@sci.osaka-cu.ac.jp (K.-i. Nakao).

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In this paper, in order to illustrate the stability problem of the superspinar, we analytically treat the linear perturbations in the near-extremal over-spinning Kerr spacetime by the manner devised by Detweiler [6–8]. It turns out that under a variety of boundary conditions the modes decay in time and the superspinar is stable.

This result may have intriguing implications on the existence and physics of very rapidly rotating compact objects in the Universe. It therefore follows from our results here that, at the very least, we need a detailed study of physically allowed boundary conditions in order to decide on the stability of superspinar or similar objects.

2. Teukolsky equations

The perturbations in the Kerr spacetime are governed by the Teukolsky equation [9]; Writing the master variable ψ in the form $\psi = e^{-i\omega t + im\varphi} S_{lm}(\theta) R_{lm}(r)$, the radial and angular Teukolsky equations are given by

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR_{lm}}{dr}\right) + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda\right)R_{lm} = 0, \tag{1}$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS_{lm}}{d\theta}\right) + \left[(a\omega\cos\theta + s)^2 - \left(\frac{m + s\cos\theta}{\sin\theta}\right)^2 - s(s-1) + F\right]S_{lm} = 0, \tag{2}$$

for the scalar ($|s| = 0$), the electromagnetic ($|s| = 1$) and gravitational ($|s| = 2$) perturbations, where $F = {}_sF^l_{m,\omega}$ with the integer l larger than or equal to $\max(|m|, |s|)$ is the separation constant equivalent to the eigenvalue of Eq. (2) with the boundary conditions of regularity at $\theta = 0$ and π, $K := (r^2 + a^2)\omega - am$, $\lambda := F + a^2\omega^2 - 2am\omega$, and $\Delta := r^2 - 2Mr + a^2$. In the case of $a^2 < M^2$, $r = r_\pm := M \pm \sqrt{M^2 - a^2}$ are real roots of $\Delta = 0$; $r = r_+$ corresponds to the event horizon and $r = r_-$ is the location of the Cauchy horizon. In the extremal case, $a^2 = M^2$, r_+ and r_- agree with each other, and there is only one degenerate event horizon. In the case of $a^2 > M^2$, i.e., the superspinar, there is no real root of $\Delta = 0$, and correspondingly no event horizon exists.

In order to see whether the superspinar is stable, we investigate the angular frequencies of the quasi-normal modes, which are linear perturbations around the Kerr metric without incoming waves at infinity. Hence we focus on the component of the Weyl tensor denoted by ψ_4, which corresponds to outgoing gravitational waves and relates to the master variable through $\psi_4 = (r - ia\cos\theta)^{-4}\,\psi$ with $s = -2$.

Hereafter, we follow Ref. [8] so that it is easy to compare the superspinar case with the black-hole case. Instead of R_{lm}, the following variable is introduced;

$$R_{lm} = \Delta^{-s}\tilde{R}_{lm}\exp\left(-i\int\frac{K}{\Delta}dr\right). \tag{3}$$

Then, Eq. (1) becomes

$$\Delta\frac{d^2\tilde{R}_{lm}}{dr^2} - \left[2i\omega(r^2 + a^2) - 2(\tilde{s}+1)(r-M) - 2iam\right] \times \frac{d\tilde{R}_{lm}}{dr} - \left[2(2\tilde{s}+1)i\omega r + \tilde{\lambda}\right]\tilde{R}_{lm} = 0, \tag{4}$$

where, using $F = E - s(s+1)$, we have introduced $\tilde{s} := -s$ and $\tilde{\lambda} := \lambda + 2s = E + a^2\omega^2 - 2am\omega - \tilde{s}(\tilde{s}+1)$.

3. Quasi-normal modes of near-extremal Kerr spacetime

We consider a near-extremal Kerr spacetime and hence we write the Kerr parameter in the form

$$a = M(1 - \epsilon),$$

assuming $0 < |\epsilon| \ll 1$. The spacetime contains a superspinar in the case of $\epsilon < 0$, whereas there is a black hole in the case of $\epsilon > 0$.

In the case of black hole, it is known that the quasi-normal mode (QNM) frequency ω approaches $m/2M$ for $m = l$ in the limit of $\epsilon \to 0_+$ [6]. The numerical study in Ref. [5] has revealed that even in the superspinar case, the QNM frequency ω approaches $m/2M$ for $m = l$ modes in the limit $\epsilon \to 0_-$. Hence, hereafter we focus on the modes of $m = l$ and assume

$$M\omega - \frac{m}{2} = \mathcal{O}\left(|\epsilon|^p\right), \tag{5}$$

where p is a positive constant.

We rewrite Eq. (4) in terms of the dimensionless variables $y := (r - M)/M$ and $\tilde{\omega} := M\omega$ as,

$$(y^2 - 2\epsilon + \epsilon^2)\frac{d^2\tilde{R}_{lm}}{dy^2} - \Big[2i\tilde{\omega}y^2 + 2(2i\tilde{\omega} - \tilde{s} - 1)y + 2i(2\tilde{\omega} - m)(1-\epsilon) + 2i\tilde{\omega}\epsilon^2\Big]\frac{d\tilde{R}_{lm}}{dy} - \left[2(2\tilde{s}+1)i\tilde{\omega}(y+1) + \tilde{\lambda}\right]\tilde{R}_{lm} = 0. \tag{6}$$

Before proceeding to our task, we briefly mention our strategy to obtain the QNM frequency for the black hole case. First, we obtain the approximate solutions of Eq. (6) in the far zone defined as $y \gg \max\left[\sqrt{|\epsilon|}, |\epsilon|^p\right]$ and the near zone defined as $y \ll 1$, separately. Then, we choose appropriate integration constants so that these solutions agree with each other in the overlapping region, $\max\left[\sqrt{|\epsilon|}, |\epsilon|^p\right] \ll y \ll 1$. Finally, we impose the no-incoming wave condition on the far-zone solution at infinity and the regularity condition on the near-zone solution at the event horizon, for black holes. A similar procedure is followed for the superspinar in order to clarify the difference from the black hole case.

In the far zone, the following equation approximates to Eq. (6);

$$y^2\frac{d^2\tilde{R}_{lm}}{dy^2} - \left[2i\tilde{\omega}y^2 + 2(2i\tilde{\omega} - \tilde{s} - 1)y\right]\frac{d\tilde{R}_{lm}}{dy} - \left[2(2\tilde{s}+1)i\tilde{\omega}(y+1) + \tilde{\lambda}\right]\tilde{R}_{lm} = 0.$$

The solution of the above equation is written in terms of confluent hypergeometric functions ${}_1F_1(\alpha; \gamma; z)$;

$$\tilde{R}^{\rm far}_{lm} = Ay^{-\tilde{s}-1/2+2i\tilde{\omega}+i\delta} \times {}_1F_1\left(\frac{1}{2} + \tilde{s} + 2i\tilde{\omega} + i\delta; 1 + 2i\delta; 2i\tilde{\omega}y\right) + By^{-\tilde{s}-1/2+2i\tilde{\omega}-i\delta} \times {}_1F_1\left(\frac{1}{2} + \tilde{s} + 2i\tilde{\omega} - i\delta; 1 - 2i\delta; 2i\tilde{\omega}y\right), \tag{7}$$

where A and B are integration constants, and

$$\delta^2 := 4\tilde{\omega}^2 - \frac{1}{4} - \tilde{\lambda} - \tilde{s}(\tilde{s}+1) \simeq \frac{1}{4}(7m^2 - 1) - E.$$

This definition of δ^2 is different from Eq. (9) in Ref. [8] due to a typo. For the near-zone analysis, we keep terms only of leading