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Maxwell–Higgs vortices with internal structure

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ABSTRACT

Vortices are considered in relativistic Maxwell–Higgs systems in interaction with a neutral scalar field. The gauge field interacts with the neutral field via the presence of generalized permeability, and the charged and neutral scalar fields interact in a way dictated by the presence of first order differential equations that solve the equations of motion. The neutral field may be seen as the source field of the vortex, and we study some possibilities, which modify the standard Maxwell–Higgs solution and include internal structure to the vortex.

1. Introduction

Vortices are planar structures that attain interesting topological behavior and have a diversity of applications in high energy physics and in condensed matter. In high energy physics in particular, in the case of a relativistic field theory, the Maxwell–Higgs model is perhaps the standard model that supports vortex configurations, as firstly shown by Nielsen and Olesen [1] and then by other researches [2–4].

The standard Maxwell–Higgs model describes an Abelian gauge field A_μ minimally coupled to a charged scalar field φ and obeys the local $U(1)$ symmetry. To develop vortex solutions, the model has to be enlarged to accommodate a potential of the Higgs type that develops spontaneous symmetry breaking. This model was long ago enlarged to accommodate the $U(1)\times U(1)$ symmetry, now with two gauge fields and two complex scalar fields that interact via an extension of the Higgs-like potential [5]. An interesting result of this model was the possibility of adding internal structure to the solution, having superconducting properties. In [6] and in the more recent works [7–9] and in references therein one finds other studies related to the presence of superconducting strings.

Another line of investigation which also deals with the $U(1)\times U(1)$ symmetry concerns the study of a visible $U(1)$ gauge field sector A_μ and another hidden $U(1)$ gauge field sector C_μ that interact via the two gauge field tensors $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and $G_{\mu\nu}=\partial_\mu C_\nu-\partial_\nu C_\mu$. The presence of the hidden sector is motivated by supersymmetric extensions of the standard model and by superstring phenomenology and may somehow play a role in the study of dark matter. Studies on vortex in such models appeared before in [10,11], and in references therein.

Recently, in [12] we started a program to describe vortex structures in generalized models in $(2,1)$ spacetime dimensions, and in [13] we studied the case of analytic vortex solutions. Other investigations on vortices that enlarge the $U(1)$ symmetry to accommodate new fields appeared before in [14–17], and more recently in [18,19] and in references therein. In particular, in [16,17] the $U(1)$ symmetry is enlarged to become $U(1)\times SO(3)$, to accommodate the $SO(3)$ spin group that under specific circumstances may lead to vortex solutions that behave as spin vortices. In this case, the $SO(3)$ symmetry is driven by the addition of neutral scalar fields that couple to the $U(1)$ symmetry via the charged Higgs-like field.

These works motivated us to go further and investigate extended versions of the generalized model. Our ultimate goal is to deal with the case in which the $U(1)\times U(1)$ symmetry plays the basic role. In the current work, however, we follow another route and take the symmetry $U(1)\times Z_2$, coupling $U(1)$ to Z_2 symmetry via the addition of a neutral scalar field, with the coupling modulated by the presence of generalized permeability. The inclusion of the Z_2 symmetry which is controlled by the neutral field is perhaps the simplest possibility to enlarge the $U(1)$ symmetry, and below we show that it may modify the profile of the vortex in a way of current interest.

2. The model

We work in $(2,1)$ flat spacetime dimensions with the Lagrangian density

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$$\mathcal{L} = -\frac{1}{4}P(\chi)F_{\mu\nu}F^{\mu\nu} + |D_\mu\varphi|^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\chi,|\varphi|) \quad (1)$$

where χ is a real scalar field, the neutral field, φ is a complex scalar field, the charged field, and A_μ is the Abelian gauge field. Also, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor and $D_\mu = \partial_\mu + ieA_\mu$ stands for the covariant derivative. The potential is denoted by $V(\chi,|\varphi|)$ and may present terms that mix the real and complex scalar fields. We suppose $P(\chi)$ is a nonnegative function of the real scalar field and use the metric tensor $\eta_{\mu\nu} = (1,-1,-1)$ and $\hbar = c = 1$. The equations of motion associated to the Lagrangian density (1) are

$$\partial_\mu\partial^\mu\chi + \frac{1}{4}P_\chi F_{\mu\nu}F^{\mu\nu} + V_\chi = 0 \quad (2a)$$

$$D_\mu D^\mu\varphi + \frac{\varphi}{2|\varphi|}V_{|\varphi|} = 0, \quad (2b)$$

$$\partial_\mu\left(PF^{\mu\nu}\right) = J^\nu, \quad (2c)$$

where the current is $J_\mu = ie(\overline{\varphi}D_\mu\varphi - \varphi\overline{D_\mu\varphi})$ and $P_\chi = dP/d\chi$, $V_\chi = \partial V/\partial\chi$, and $V_{|\varphi|} = \partial V/\partial|\varphi|$. By setting $\nu = 0$ in equation (2c), one can show that for static field configurations the Gauss' law is satisfied with $A_0 = 0$. In this case, the vortex is electrically neutral since its electric charge vanishes.

To search for topological solutions, we consider static configurations and suppose that

$$\chi = \chi(r), \quad \varphi = g(r)e^{in\theta}, \quad \vec{A} = -\frac{\hat{\theta}}{er}(a(r) - n), \quad (3)$$

in which $n \in \mathbb{Z}$ is the vorticity. The functions $\chi(r)$, $a(r)$ and $g(r)$ obey the boundary conditions

$$\chi(0) = \chi_0, \qquad g(0) = 0, \qquad a(0) = n, \quad (4)$$

$$\chi(\infty) = \chi_\infty, \qquad g(\infty) = v, \qquad a(\infty) = 0. \quad (5)$$

Here, χ_0, χ_∞ and v are parameters involved in the symmetry breaking of the potential. Considering the fields described by equations (3), the magnetic field has to satisfy

$$B = -F^{12} = -\frac{a'}{er}, \quad (6)$$

where the prime stands for the derivative with respect to r. By using this, one can show the magnetic flux is quantized

$$\Phi = 2\pi\int rdrB = \frac{2\pi n}{e}. \quad (7)$$

The equations of motion (2) with the static fields (3) assume the form

$$\frac{1}{r}\left(r\chi'\right)' = P_\chi\frac{a'^2}{2e^2r^2} + V_\chi, \quad (8a)$$

$$\frac{1}{r}\left(rg'\right)' = \frac{a^2g}{r^2} + \frac{1}{2}V_{|\varphi|}, \quad (8b)$$

$$r\left(P\frac{a'}{er}\right)' = 2eag^2. \quad (8c)$$

The energy density for static field configurations can be calculated standardly; one uses (3) to write

$$\rho = P\frac{a'^2}{2e^2r^2} + g'^2 + \frac{a^2g^2}{r^2} + \frac{1}{2}\chi'^2 + V. \quad (9)$$

The equations of motion (8) are of second order and present couplings between the fields. In order to get first order equations, we use the Bogomol'nyi procedure [2] and introduce an auxiliary function $W = W(\chi)$ to write the energy density (9) as

$$\begin{aligned}\rho = {} & \frac{P(\chi)}{2}\left(\frac{a'}{er} \pm \frac{e(v^2-g^2)}{P(\chi)}\right)^2 + \left(g' \mp \frac{ag}{r}\right)^2 \\ & + \frac{1}{2}\left(\chi' \mp \frac{W_\chi}{r}\right)^2 + V - \left(\frac{e^2}{2}\frac{\left(v^2-g^2\right)^2}{P(\chi)} + \frac{1}{2}\frac{W_\chi^2}{r^2}\right) \\ & \pm\frac{1}{r}\left(W - a\left(v^2-g^2\right)\right)',\end{aligned} \quad (10)$$

where $W_\chi = dW/d\chi$. If the potential is written as

$$V(\chi,|\varphi|) = \frac{e^2}{2}\frac{\left(v^2-|\varphi|^2\right)^2}{P(\chi)} + \frac{1}{2}\frac{W_\chi^2}{r^2}, \quad (11)$$

the energy becomes

$$\begin{aligned}E = {} & 2\pi\int_0^\infty r\,dr\,\frac{P(\chi)}{2}\left(\frac{a'}{er} \pm \frac{e(v^2-g^2)}{P(\chi)}\right)^2 \\ & + 2\pi\int_0^\infty r\,dr\left(g' \mp \frac{ag}{r}\right)^2 \\ & + 2\pi\int_0^\infty r\,dr\,\frac{1}{2}\left(\chi' \mp \frac{W_\chi}{r}\right)^2 + E_B,\end{aligned} \quad (12)$$

where

$$\begin{aligned}E_B &= \pm 2\pi\int_0^\infty dr\left(W - a\left(v^2-g^2\right)\right)' \\ &= 2\pi\left|W(\chi(\infty)) - W(\chi(0))\right| + 2\pi|n|v^2.\end{aligned} \quad (13)$$

Since the three integrands in the energy (12) are all non-negative, we see that the energy is bounded by E_B, i.e., $E \geq E_B$. If the solutions obey the equations

$$\chi' = \pm\frac{W_\chi}{r} \quad (14)$$

and

$$g' = \pm\frac{ag}{r}, \quad (15a)$$

$$-\frac{a'}{er} = \pm\frac{e\left(v^2-g^2\right)}{P(\chi)}, \quad (15b)$$

the Bogomol'nyi bound is saturated, such that the energy is minimized to $E = E_B$. Therefore, we have obtained three first order equations to study the problem, since they satisfy the equations of motion (8). As one knows, the fact that the solutions of the above first order equations (14) and (15) saturate the Bogomol'nyi bound implies stability against decay into similar lower energy configurations.

It is worth commenting that the equation for the real scalar field (14) does not depend on the other fields. Thus, the real scalar field can be seen as a source to generate the vortex configuration, and we call it the source field. Although this is not apparent from the equations of motion (8), it is clear in the first order equations. Moreover, concerning the first order equations, it seems that the model one is dealing with is the bosonic portion of a larger, supersymmetric theory, which will be further investigated elsewhere. Here we keep working with the above model, since it unveils several interesting possibilities of investigations of current interest.