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Extended Reissner–Nordström solutions sourced by dynamical torsion

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ABSTRACT

We find a new exact vacuum solution in the framework of the Poincaré Gauge field theory with massive torsion. In this model, torsion operates as an independent field and introduces corrections to the vacuum structure present in General Relativity. The new static and spherically symmetric configuration shows a Reissner–Nordström-like geometry characterized by a spin charge. It extends the known massless torsion solution to the massive case. The corresponding Reissner–Nordström–de Sitter solution is also compatible with a cosmological constant and additional $U(1)$ gauge fields.

1. Introduction

The fundamental relation of the energy and momentum of matter with the space–time geometry is one of the most important foundations of General Relativity (GR). Namely, the energy-momentum tensor acts as the source of gravity, which is appropriately described in terms of the curvature tensor. In an analogous way, it may be expected that the intrinsic angular momentum of matter may also act as an additional source of the interaction and extend such a geometrical scheme.

Poincaré Gauge (PG) theory of gravity is the most consistent extension of GR that provides a suitable correspondence between spin and the space–time geometry by assuming an asymmetric affine connection defined within a Riemann–Cartan (RC) manifold (i.e. endowed with curvature and torsion) [1,2]. It represents a gauge approach to gravity based on the semidirect product of the Lorentz group and the space–time translations, in analogy to the unitary irreducible representations of relativistic particles labeled by their spin and mass, respectively. Then not only an energy-momentum tensor of matter arises from this approach, but also a non-trivial spin density tensor that operates as source of torsion and allows the existence of a gravitating antisymmetric component of the former, which may induce changes in the geometrical structure of the space–time, as the rest of the components of the mentioned tensor. This fact contrasts with the established by GR, where all the possible geometrical effects occurred in the Universe can be only provided by a symmetric component of the energy-momentum tensor, despite the existence of dynamical configurations endowed with asymmetric energy-momentum tensors [3,4].

Accordingly, a gauge invariant Lagrangian can be constructed from the field strength tensors to introduce the extended dynamical effects of the gravitational field. In this sense, it is well-known that the role of torsion depends on the order of the mentioned field strength tensors present in the Lagrangian, in a form that only quadratic or higher order corrections in the curvature tensor involve the presence of a non-trivial dynamical torsion, whose effects can propagate even in a vacuum space–time.

Likewise, the distinct restrictions on the Lagrangian parameters lead to a large class of gravitational models where an extensive number of particular and fundamental differences may arise. For example, in analogy to the standard approach of GR, it was shown that the Birkhoff's theorem is satisfied only in certain cases of the PG theory [5,6]. Indeed, the dynamical role of the new degrees of freedom involved in such a theory can modify the space–time geometry and even predominate in their respective domains of applicability. The

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search and study of exact solutions are therefore essential in order to improve the understanding and physical interpretation of the new framework.

A large class of exact solutions have been found since the formulation of the theory, especially for the case of static and spherically symmetric vacuum space–times, where one of the most primary and remarkable solutions is the so called Baekler solution, associated with a sort of PG models that encompass a weak-field limit with an additional confinement type of potential besides the Newtonian one [7], giving rise to a Schwarzschild–de Sitter geometry in analogy to the effect caused by the presence of a cosmological constant in the regular gravity action [8]. Furthermore, additional results have also been systematically obtained for a large class of PG configurations, such as axisymmetric space–times, cosmological systems or generalized gravitational waves (see [2,9–11] and references therein).

Recently, the authors of this work found a new exact solution with massless torsion associated with a PG model containing higher order corrections quadratic in the curvature tensor, in such a way that the standard framework of GR is naturally recovered when the total curvature satisfies the first Bianchi identity of the latter. This construction ensures that all the new propagating degrees of freedom introduced by the model fall on the torsion field, so that this quantity extend the domain of applicability of the standard case. Thus, it was shown that the regular Schwarzschild geometry provided by the Birkhoff's theorem of GR can be replaced by a Reissner–Nordström (RN) space–time with RC Coulomb-like curvature when this sort of dynamical torsion is considered [12]. This result contrasts with other post-Riemannian solutions, such as the derived in the framework of the Metric-Affine Gauge (MAG) theory, where the non-metricity tensor can involve an analogous vacuum RN configuration [13,14]. In addition, it is reasonable to expect that such a configuration may be extended for the case where additional non-vanishing mass modes of the torsion tensor are present in the Lagrangian, in order to analyze the equivalent PG model with massive torsion. As we will show, we have found the associated RN solution with massive torsion and generalized the previous approach according to the scheme performed in that simpler case.

This paper is organized as follows. First, in Section 2, we introduce our PG model with massive torsion and briefly describe its general mathematical foundations. The analysis and application of the resulting field equations in the static spherically symmetric space–time is shown in Section 3, in order to find the appropriate vacuum solutions for the selected case. In section 4, we present the required new PG solution with massive torsion and extend our previous results related to the massless case. We present the conclusions of our work in Section 5. Finally, we detail in Appendix A the geometrical quantities involved in the vacuum field equations associated with this model.

Before proceeding to the main discussion and general results, we briefly introduce the notation and physical units to be used throughout this article. Latin a, b and greek μ, ν indices refer to anholonomic and coordinate basis, respectively. We use notation with tilde for magnitudes including torsion and without tilde for torsion-free quantities. On the other hand, we will denote as P_a the generators of the space–time translations as well as J_{ab} the generators of the space–time rotations and assume their following commutative relations:

$$[P_a, P_b] = 0\,, \tag{1}$$

$$[P_a, J_{bc}] = i\,\eta_{a[b}\,P_{c]}\,, \tag{2}$$

$$[J_{ab}, J_{cd}] = \frac{i}{2}\,(\eta_{ad}\,J_{bc} + \eta_{cb}\,J_{ad} - \eta_{db}\,J_{ac} - \eta_{ac}\,J_{bd})\,. \tag{3}$$

Finally, we will use Planck units ($G = c = \hbar = 1$) throughout this work.

2. Quadratic Poincaré gauge gravity model with massive torsion

We start from the general gravitational action associated with our original PG model and incorporate the three independent quadratic scalar invariants of torsion into this expression, which represent the mass terms of the mentioned quantity:

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big[\mathcal{L}_m - \tilde{R} - \frac{1}{4}\,(d_1+d_2)\,\tilde{R}^2 - \frac{1}{4}\,(d_1+d_2+4c_1+2c_2)\,\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\,\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
$$+c_2\,\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} + d_1\,\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} + d_2\,\tilde{R}_{\mu\nu}\tilde{R}^{\nu\mu} + \alpha\,T_{\lambda\mu\nu}T^{\lambda\mu\nu} + \beta\,T_{\lambda\mu\nu}T^{\mu\lambda\nu} + \gamma\,T^{\lambda}{}_{\lambda\nu}T^{\mu}{}_{\mu}{}^{\nu}\Big]\,, \tag{4}$$

where $c_1, c_2, d_1, d_2, \alpha, \beta$ and γ are constant parameters.

The field strength tensors above derive from the gauge connection of the Poincaré group $ISO(1,3)$, which can be expressed in terms of the generators of translations and local Lorentz rotations in the following way:

$$A_\mu = e^a{}_\mu\,P_a + \omega^{ab}{}_\mu\,J_{ab}\,, \tag{5}$$

where $e^a{}_\mu$ is the vierbein field and $\omega^{ab}{}_\mu$ the spin connection of a RC manifold, related to the metric tensor and the metric-compatible affine connection as usual [15]:

$$g_{\mu\nu} = e^a{}_\mu\,e^b{}_\nu\,\eta_{ab}\,, \tag{6}$$

$$\omega^{ab}{}_\mu = e^a{}_\lambda\,e^{b\rho}\,\tilde{\Gamma}^\lambda{}_{\rho\mu} + e^a{}_\lambda\,\partial_\mu\,e^{b\lambda}\,. \tag{7}$$

The affine connection is decomposed into the torsion-free Levi-Civita connection and a contortion component, which transforms as a tensor due to the tensorial nature of torsion since it describes the antisymmetric part of the affine connection:

$$\tilde{\Gamma}^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} + K^\lambda{}_{\mu\nu}\,. \tag{8}$$

Thus, the presence of torsion potentially introduces changes in the properties of the gravitational interaction and it involves the following $ISO(1,3)$ gauge field strength tensors: