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(Compactified) black branes in four dimensional $f(R)$-gravity

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ABSTRACT

A new family of analytical solutions in a four dimensional static spacetime is presented for $f(R)$-gravity. In contrast to General Relativity, we find that a non trivial black brane/string solution is supported in vacuum power law $f(R)$-gravity for appropriate values of the parameters characterizing the model and when axisymmetry is introduced in the line element. For the aforementioned solution, we perform a brief investigation over its basic thermodynamic quantities.

1. Introduction

Modified theories of gravity have drawn the attention of the scientific community because of the geometric mechanics that they provide to describe various phenomena in nature. In this concept, new geometrodynamical degrees of freedom are introduced in the gravitational field equations in such a way so as to modify Einstein's General Relativity (GR). These additional terms can have either theoretical or phenomenological origin [1]. Among the various proposed modified theories of gravity, $f(R)$-gravity [2] has been the main subject of study in various works over different areas of gravitational physics.

$f(R)$-gravity is a fourth-order theory, where the action integral involves a function of the spacetime scalar curvature R. General Relativity, with or without cosmological constant, is the special limit of $f(R)$-gravity when the theory becomes of second-order; that is, when f is a linear function of the Ricci scalar. It belongs to a more general family of theories that take into account curvature terms in order to modify the Einstein–Hilbert action [3–6]. The gravitational field equations of $f(R)$-gravity are dynamically equivalent to that of O'Hanlon theory [7], where a Lagrange multiplier is introduced so as to reduce the order of the theory by increasing the number of degrees of freedom through the introduction of a scalar field [8,9]. This scalar field is nonminimally coupled to gravity and recovers Brans–Dicke theory [10] with a zero Brans–Dicke parameter. Hence, $f(R)$-gravity is also related to families of Horndeski theories [11], which means that it is free of Ostrogradsky's instabilities [12,13]. For a recent discussion on the correspondence among $f(R)$-gravity and other theories through the various frames, together with relevant implications on conservation laws, see [14,15].

As we mentioned before, the applications of $f(R)$-gravity in gravitational physics cover various subjects. As far as cosmology is concerned, the theory is used both to model the inflationary phase of the universe [16–22] and also as a dark energy candidate to describe the late-time acceleration phase [23–31]. In [32] it was found that new Kasner-like solutions exist, while some cosmological solutions in locally rotational spacetimes were derived in [33,34]. Some effects on the Mixmaster universe can be found in [35,36]. In general, the various implications of $f(R)$-gravity – and of other theories of gravitation as well – from a cosmological perspective can be seen in [37,38]. Other studies on gravitational collapse can be encountered in [39,40], while static spherically solutions were derived in [41–43] with or without a constant Ricci scalar. Black hole solutions have been also investigated in the literature, for instance see [44–51] and references therein, as also physical phenomena like, binary black hole merge [52], anti-evaporation [53], black hole thermodynamics [54,55] and many other.

It is well known that black string solutions can be easily constructed in Einstein's gravity by trivially embedding black hole solutions in higher dimensions. The same is also true for cer-

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tain classes of modified theories of gravitation (for example it holds for a certain type of Lovelock Lagrangians [56,57]). In many cases, numerical solutions have been presented in the literature [58–60]. However, exact solutions are always of special interest and there exists an extended bibliography over the subject covering a great number of gravitational configurations: from three dimensional charged black strings [61], to cosmological constant solutions in an arbitrary number of dimensions [62], even in the presence of axionic scalar fields [63]. Some exact solutions which describe rotating black strings in $f(R)$-gravity in the presence of a electromagnetic field were derived in [64], while some asymptotic black strings solutions can be found in [65] and a cosmic sting solution in four dimensions in the context of scalar tensor theory has been given in [66]. The stability of black string solutions is always an issue, since in general these geometries are unstable, see for example [67–69]. However, counterexamples of this general rule exist and stable solutions may also arise [73,74]. In what regards other interesting gravitational solutions, a toroidal black hole has also emerged in the Einstein nonlinear sigma model [70]. What is more, in the literature one can also find brane solutions in higher dimensional $f(R)$-gravity [71,72].

In this work we start by investigating analytical solutions of power law $f(R)$-gravity in a four-dimensional static spacetime. In this context we derive the general analytical solution and try to see under which conditions interesting gravitational objects may be described by it. We find that a black brane/string solution can be distinguished for certain values of the involved parameters in the case where the metric is axisymmetric. The outline of the paper is as follows: In Section 2, we briefly discuss $f(R)$-gravity and derive the field equations for the spacetime of our consideration. In Section 3, we obtain the general solution for the induced system of equations. Section 4 includes the main results of our analysis which is the black brane/string solution of the field equations for the power-law theory $f(R)=R^k$. In Section 5, we calculate the surface gravity and thus the temperature of the system as well as the entropy on the horizon. Finally, in Section 6, we discuss our results and draw our conclusions.

2. Preliminaries

The action integral of $f(R)$-gravity constitutes a modification of the Einstein–Hilbert action and is given by the following expression

$$\mathcal{S}=\int dx^4\sqrt{-g}\,f(R)\,, \tag{1}$$

where R is the Ricci scalar constructed from the spacetime metric $g_{\mu\nu}$. It follows from (1) that the field equations of Einstein's GR are recovered when $f(R)$ is a linear function of R.

Variation with respect to the metric tensor gives the field equations

$$f_{,R}R_{\mu\nu}-\frac{1}{2}fg_{\mu\nu}-\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla_\sigma\nabla^\sigma\right)f_{,R}=0, \tag{2}$$

where we have assumed that we are in vacuum, i.e. there does not exist any matter source. The Ricci scalar contains second order derivatives of the coefficients of the metric $g_{\mu\nu}$, hence, relation (2) provides a fourth-order system of differential equations.

An alternative way to write the latter is by the use of Einstein's tensor together with the definition of an energy-momentum tensor of geometric origin. In particular, we can rewrite (2) as

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=k_{eff}T^{eff}_{\mu\nu}\,, \tag{3}$$

where $T^{eff}_{\mu\nu}$ is the effective energy momentum tensor that includes the terms which make the theory deviate from General Relativity,

$$T^{eff}_{\mu\nu}=\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla_\sigma\nabla^\sigma\right)f_{,R}+\frac{1}{2}\left(f-Rf_{,R}\right)g_{\mu\nu} \tag{4}$$

while $k_{eff}=\left(f_{,R}(R)\right)^{-1}$ is a varying gravitational constant. From the latter it follows that the theory is defined in the Jordan frame.

Apart from the limit in which $f_{,RR}(R)=0$, where General Relativity is recovered, it can be observed that any constant Ricci curvature ($R=R_0$) solution of General Relativity also satisfies field equations (3) if the following algebraic condition relating the free parameters of the $f(R)$ function to R_0 [75] holds

$$2f(R_0)-R_0f_{,R}(R_0)=0. \tag{5}$$

In our work we assume the following static metric with line element

$$ds^2=-a(r)^2dt^2+N(r)^2dr^2+b(r)^2d\phi^2+c(r)^2d\zeta^2 \tag{6}$$

It is interesting to note that, when $b(r)=c(r)$ and the line element is invariant under rotations in the $\zeta-\phi$ plane, the general solution of a geometry characterized by a constant Ricci scalar $R_0\neq 0$ is

$$ds^2=\mp\left(r^2-\frac{1}{r}\right)dt^2+\frac{12}{R_0}\left(\frac{1}{r}-r^2\right)^{-1}dr^2+r^2\left(d\phi^2+d\zeta^2\right), \tag{7}$$

which – as we noted earlier – is also a solution of GR in the presence of a cosmological constant. Line element (7) can be also seen to be a special case of a more general solution presented in [76] including also an electromagnetic field. Clearly, the minus branch of (7) characterizes a black brane or string (depending on the topology of the (ϕ,ζ) surface) when $R_0<0$. For $f(R)$-gravity the constant scalar curvature is related to the parameters of the relative model through algebraic equation (5).

The existence of a solution like (7) is our motive to start investigating a more general setting given by line element (6). Thus, we begin by considering $b(r)\neq c(r)$ and turn to the general case where R is not a constant. The Ricci scalar is calculated to be[1]

$$R=-\frac{2}{N^2}\left(\frac{a''}{a}+\frac{b''}{b}+\frac{c''}{c}+\frac{a'b'}{ab}+\frac{a'c'}{ac}+\frac{b'c'}{bc}+\right)+2\frac{N'}{N^3}\left(\frac{a'}{a}+\frac{b'}{b}+\frac{c'}{c}\right). \tag{8}$$

and the gravitational field equations (2) are

$$R'^2f_{,RRR}+\left[\left(\frac{b'}{b}+\frac{c'}{c}-\frac{N'}{N}\right)R'+R''\right]f_{,RR}-\left(\frac{a''}{a}+\frac{a'b'}{ab}+\frac{a'c'}{ac}-\frac{a'N'}{aN}\right)f_{,R}-\frac{1}{2}N^2f=0 \tag{9}$$

$$R'\left(\frac{a'}{a}+\frac{b'}{b}+\frac{c'}{c}\right)f_{,RR}-\left(\frac{a''}{a}+\frac{b''}{b}+\frac{c''}{c}-\frac{a'N'}{aN}-\frac{b'N'}{bN}-\frac{c'N'}{cN}\right)f_{,R}-\frac{1}{2}N^2f=0 \tag{10}$$

$$R'^2f_{,RRR}+\left[\left(\frac{a'}{a}+\frac{c'}{c}-\frac{N'}{N}\right)R'+R''\right]f_{,RR}-\left(\frac{b''}{b}+\frac{a'b'}{ab}+\frac{b'c'}{bc}-\frac{b'N'}{bN}\right)f_{,R}-\frac{1}{2}N^2f=0 \tag{11}$$

[1] The prime "′" denotes a total derivative with respect to the variable r, that is $a'(r)=\frac{da(r)}{dr}$.