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Quasi-normal modes of holographic system with Weyl correction and momentum dissipation

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ABSTRACT

We study the charge response in complex frequency plane and the quasi-normal modes (QNMs) of the boundary quantum field theory with momentum dissipation dual to a probe generalized Maxwell system with Weyl correction. When the strength of the momentum dissipation $\hat{\alpha}$ is small, the pole structure of the conductivity is similar to the case without the momentum dissipation. The qualitative correspondence between the poles of the real part of the conductivity of the original theory and the ones of its electromagnetic (EM) dual theory approximately holds when $\gamma \to -\gamma$ with γ being the Weyl coupling parameter. While the strong momentum dissipation alters the pole structure such that most of the poles locate at the purely imaginary axis. At this moment, the correspondence between the poles of the original theory and its EM dual one is violated when $\gamma \to -\gamma$. In addition, for the dominant pole, the EM duality almost holds when $\gamma \to -\gamma$ for all $\hat{\alpha}$ except for a small region of $\hat{\alpha}$.

1. Introduction

The quantum critical (QC) dynamics is a long-standing important issues in strongly coupling condensed matter systems, in which the perturbative techniques in traditional field theory unfortunately lose its power. An alternative method is the AdS/CFT correspondence [1–4], which maps a strongly coupled quantum field theory to a weakly coupled gravitational theory in the large N limit. By this way, the holographic QC dynamics at zero density, dual to a probe Maxwell field coupled to the Weyl tensor $C_{\mu\nu\rho\sigma}$ in the Schwarzschild-AdS (SS-AdS) in bulk, has been widely studied in [5–12]. Since the Weyl tensor is taken into account, it exhibits non-trivial frequency dependent conductivity. In particular, depending on the sign of the coupling parameter γ, a Damle–Sachdev (DS) peak [13] resembling the particle response or a dip resembling the vortex response is observed in optical conductivity[1] [5]. It is analogous to that of the superfluid-insulator quantum critical point (QCP) [5–7]. Further, the charge response from higher derivative (HD) theory is studied, in which we have an arbitrarily sharp Drude-like peak or vanishing DC conductivity depending on the coupling parameter [16]. Of particular interest is that the behavior of the holographic HD system is very similar to the $O(N)$ $NL\sigma M$ model for large-N [13]. Another important progress is the construction of neutral scalar hair black brane in bulk by introducing the coupling between Weyl tensor and neutral scalar field, which is dual to QC dynamics and the one away from QCP in the boundary field theory [17,18].

Also, we can introduce the momentum dissipation, implemented by a pair of axionic fields linearly dependent on spatial coordinates [34], into the holographic QC systems studied in [5,6, 8–12], which is away from QCP, to explore the corresponding effects. We observe that for the 4 derivative theory, the momentum dissipation drives the peak and dip into each other [19], while for the 6 derivative theory, similar behaviors are not observed [20]. Another appealing phenomena for 4 derivative theory is that there is a specific value of the momentum dissipation strength, i.e., $\hat{\alpha} = 2/\sqrt{3}$, for which the particle-vortex duality exactly remains [19]. In this paper, we want to extend the previous works [19] to the charge response in complex frequency plane. We shall

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[1] Those kind of peak and dip features have also been observed in probe branes setups [14] and just higher terms in F^2 with F being the Maxwell field strength [15]. Although the role and the presence of a Drude-like peak in the optical conductivity have been partially explained for the probe brane case [14], it is still not so clear why there is a Drude-like peak and to what it is connected. More efforts and attempts are deserved to further pursuit in future.

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particularly focus on the properties of the quasi-normal modes (QNMs), which corresponds to the poles of the retarded Green's function for the dual boundary CFT (see [21–33] and references therein). Our paper is organized as follows. In Section 2, we introduce the holographic framework, including the Einstein-axions (EA) theory, which is the background geometry dual to a specific thermal excited state with momentum dissipation, and the 4 derivative theory without electromagnetic (EM) duality, which is regarded as the probe on top of the EA background geometry. And then, we calculate the conductivity in the complex frequency plane in Section 3. We are in particular interested in the QNMs of our present models, which are presented in Section 4. The conclusions and discussions are summarized in Section 5.

2. Holographic framework

We consider a specific thermal state with homogeneous disorder, which is holographically described by the EA theory [34] (also refer to [35–41] and references therein)

$$S_0 = \int d^4x\sqrt{-g}\Big(R + 6 - \frac{1}{2}\sum_{I=x,y}(\partial\phi_I)^2\Big), \tag{1}$$

where $\phi_I = \alpha x_I$ with $I = x, y$ and α being a constant. In this action, there is a negative cosmological constant $\Lambda = -6$, which supports an asymptotically AdS spacetimes.[2] The EA action (1) gives a neutral black brane solution [34]

$$ds^2 = \frac{1}{u^2}\Big(-f(u)dt^2 + \frac{1}{f(u)}du^2 + dx^2 + dy^2\Big),$$
$$f(u) = (1-u)p(u), \qquad p(u) = \frac{\sqrt{1+6\hat{\alpha}^2} - 2\hat{\alpha}^2 - 1}{\hat{\alpha}^2}u^2 + u + 1. \tag{2}$$

$u = 0$ is the asymptotically AdS boundary while the horizon locates at $u = 1$. Here we have parameterized the black brane solution by $\hat{\alpha} = \alpha/4\pi T$ with the Hawking temperature $T = p(1)/4\pi$. Note that for the particular way we adopt to break translations, i.e., the axionic fields, the original (spacetime translations) $\times$ (internal translations) group is broken to the diagonal subgroup such that the geometry is homogeneous [42,43].

On top of the geometry background (2), we consider the following 4 derivative theory [5] (also see [9–12,16,19,20])

$$S_A = \int d^4x\sqrt{-g}\Big(-\frac{1}{8g_F^2}F_{\mu\nu}X^{\mu\nu\rho\sigma}F_{\rho\sigma}\Big), \tag{3}$$

where

$$X_{\mu\nu}^{\ \ \rho\sigma} = I_{\mu\nu}^{\ \ \rho\sigma} - 8\gamma C_{\mu\nu}^{\ \ \rho\sigma}, \qquad I_{\mu\nu}^{\ \ \rho\sigma} = \delta_\mu^{\ \rho}\delta_\nu^{\ \sigma} - \delta_\mu^{\ \sigma}\delta_\nu^{\ \rho}. \tag{4}$$

The new tensor X in the above equation possess the following symmetries

$$X_{\mu\nu\rho\sigma} = X_{[\mu\nu][\rho\sigma]} = X_{\rho\sigma\mu\nu}. \tag{5}$$

When we set $X_{\mu\nu}^{\ \ \rho\sigma} = I_{\mu\nu}^{\ \ \rho\sigma}$, the generalized Maxwell theory (3) reduces to the standard version. And then, we can write down the equation of motion from the action (3),

$$\nabla_\nu(X^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0. \tag{6}$$

Next, we can construct the corresponding dual EM theory, which is [5]

$$S_B = \int d^4x\sqrt{-g}\Big(-\frac{1}{8\hat{g}_F}G_{\mu\nu}\widehat{X}^{\mu\nu\rho\sigma}G_{\rho\sigma}\Big), \tag{7}$$

where $\hat{g}_F^2 \equiv 1/g_F^2$ and $G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ is the dual field strength. The tensor $\widehat{X}$ satisfies

$$\widehat{X}_{\mu\nu}^{\ \ \rho\sigma} = -\frac{1}{4}\varepsilon_{\mu\nu}^{\ \ \alpha\beta}(X^{-1})_{\alpha\beta}^{\ \ \gamma\lambda}\varepsilon_{\gamma\lambda}^{\ \ \rho\sigma}, \qquad \frac{1}{2}(X^{-1})_{\mu\nu}^{\ \ \rho\sigma}X_{\rho\sigma}^{\ \ \alpha\beta} \equiv I_{\mu\nu}^{\ \ \alpha\beta}. \tag{8}$$

And then the equation of motion of the dual theory (7) can be written as

$$\nabla_\nu(\widehat{X}^{\mu\nu\rho\sigma}G_{\rho\sigma}) = 0. \tag{9}$$

For the standard Maxwell theory in four dimensional bulk spacetimes, $\widehat{X}_{\mu\nu}^{\ \ \rho\sigma} = I_{\mu\nu}^{\ \ \rho\sigma}$, which indicates that both the theories (3) and (7) are identical and so the Maxwell theory is self-dual. The coupling term between the Weyl tensor and the Maxwell field strength breaks such self-duality. But for small γ, since

$$(X^{-1})_{\mu\nu}^{\ \ \rho\sigma} = I_{\mu\nu}^{\ \ \rho\sigma} + 8\gamma C_{\mu\nu}^{\ \ \rho\sigma} + \mathcal{O}(\gamma^2), \tag{10}$$
$$\widehat{X}_{\mu\nu}^{\ \ \rho\sigma} = (X^{-1})_{\mu\nu}^{\ \ \rho\sigma} + \mathcal{O}(\gamma^2) \tag{11}$$

we have an approximate duality between the theories (3) and (7) with the change of the sign of γ.

3. Conductivity in the complex frequency plane

In our previous works [19], the conductivity on the real frequency axis was numerically studied. Here, we extend the study from real frequency axis to complex frequency plane, $\omega \equiv \mathrm{Re}\omega + i\,\mathrm{Im}\omega \in \mathbb{C}$. To this end, we turn on the perturbation of the gauge field along y direction like $A_y(t,u) \sim e^{-i\omega t}A_y(u)$. And then, the perturbative equation can be written down [5,19]

$$A_y'' + \Big(\frac{f'}{f} + \frac{X_6'}{X_6}\Big)A_y' + \frac{\mathfrak{p}^2\hat{\omega}^2}{f^2}\frac{X_2}{X_6}A_y = 0, \tag{12}$$

where X_i, $i = 1,\ldots,6$, are the components of $X_A^{\ B}$ defined as $X_A^{\ B} = \{X_1(u), X_2(u), X_3(u), X_4(u), X_5(u), X_6(u)\}$, with $A, B \in \{tx, ty, tu, xy, xu, yu\}$. Note that $X_1(u) = X_2(u)$ and $X_5(u) = X_6(u)$ due to the isotropy. In addition, the dimensionless frequency $\hat{\omega} \equiv \frac{\omega}{4\pi T} = \frac{\omega}{\mathfrak{p}}$, with $\mathfrak{p} \equiv p(1) = 4\pi T$, has been introduced in the perturbative equation (12). Next, we can numerically solve the Eq. (12) with the ingoing condition at horizon on top of complex frequency plan and read off the optical conductivity in terms of

$$\sigma(\omega) = \frac{\partial_u A_y(u,\hat{\omega})}{i\omega A_y(u,\hat{\omega})}. \tag{13}$$

By letting $A_\mu \to B_\mu$ and $X_i \to \widehat{X}_i = 1/X_i$, the corresponding equation of motion for the dual theory can be obtained.

The numerical results are shown in Fig. 1, 2, 3 and 4 for the representative $\hat{\alpha}$ and the two values of γ satisfying the bound, $\gamma = \pm 1/12$. We find that all the poles of the conductivity are in the lower-half plane (LHP), which indicates that the perturbative modes are stable ones. As a check on our numerics, we show the conductivity σ (left panels) and its dual σ_* (right panels) in the LHP for $|\gamma| = 1/12$ (the panels above are for $\gamma = 1/12$ and the ones below for $\gamma = -1/12$) and $\hat{\alpha} = 0$ in the complex frequency plane, which have been obtained in [9]. Our results are in excellent agreement with that in [9]. For comparison, we firstly summary the main properties of the pole structure as what follows.

[2] Here, without loss of generality we have set the AdS radius $L = 1$ for simplicity.