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Color superconductivity from the chiral quark–meson model

Armen Sedrakian [a,*], Ralf-Arno Tripolt [b], Jochen Wambach [b,c]

[a] Frankfurt Institute for Advanced Studies, Ruth-Moufang-str. 1, D-60438 Frankfurt-Main, Germany
[b] European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler, Villa Tambosi, Strade delle Tabarelle 286, I-38123 Villazzano (TN), Italy
[c] Theoriezentrum, Institut für Kernphysik, Technische Universität Darmstadt, Schlossgarten-str. 2, 64289 Darmstadt, Germany

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ABSTRACT

We study the two-flavor color superconductivity of low-temperature quark matter in the vicinity of chiral phase transition in the quark–meson model where the interactions between quarks are generated by pion and sigma exchanges. Starting from the Nambu–Gorkov propagator in real-time formulation we obtain finite temperature (real axis) Eliashberg-type equations for the quark self-energies (gap functions) in terms of the in-medium spectral function of mesons. Exact numerical solutions of the coupled nonlinear integral equations for the real and imaginary parts of the gap function are obtained in the zero temperature limit using a model input spectral function. We find that these components of the gap display a complicated structure with the real part being strongly suppressed above $2\Delta_0$, where Δ_0 is its on-shell value. We find $\Delta_0 \simeq 40$ MeV close to the chiral phase transition.

1. Introduction

Low-temperature quark matter at large chemical potential is expected to be a color superconductor [1,2]. In its ground state, it forms a coherent state of bound Cooper pairs which flow without resistivity. At moderate densities, the most robust pairing pattern involves two light flavors of up and down quarks forming Cooper pairs with a wave-function that is antisymmetric in color space [3].

Experimental programs exploring highly compressed matter in heavy-ion collisions will probe the region of the phase diagram of strong interaction matter where the interplay between the chiral symmetry breaking and color superconductivity is an important factor [4]. In this regime of interest, which is close to the chiral phase transition line, quarks and mesons are the dominant degrees of freedom. Having this context in mind, we address here the 2SC pairing in quark matter in the quark–meson model, which is a renormalizable model that shares the chiral symmetry breaking pattern with the underlying fundamental theory of QCD [5,6]. More specifically, our work is further motivated by the recent observation that the entropy of this model shows anomalies at low-temperatures, when studied within the functional renormalization group formalism [7]. This could be an indication of the instability of the obtained ground state toward color superconductivity or some other phase of QCD, for example, the quarkyonic phase [8].

Color superconductivity in the 2SC phase was studied at asymptotically high densities within perturbative QCD framework in Refs. [9–12]. In these theories, the interaction between quarks is mediated via (screened) gluon exchanges and the pairing fields are governed by Eliashberg-type equations, familiar from boson-exchange models of superconductivity. Approximate solutions of these equations for the case of massless quarks were obtained which exhibit the scaling of the gap (more precisely, its on-shell value Δ_0) with the strong coupling λ as $\Delta_0 \propto \exp(-1/\lambda)$; these solutions also identified the pre-factor of the (approximate) gap equation for the real part of the pairing field. However, to our knowledge, the effects of retardation of interaction via gluon or other exchanges and the resulting complex nature of the gap function have not been exposed so far.

The aim of this work is thus to address again the problem of 2SC pairing, however within a model which is better suited in the regime close to the chiral phase transition and to maintain the complex nature of the gap throughout the calculation. We choose to work with the quark–meson model, where the interaction between quarks is mediated by pseudo-scalar pion exchanges and scalar sigma exchanges. The quarks are assumed to be massive due to the dynamical mechanism of chiral symmetry breaking. We find the equations for the 2SC pairing gap appropriate for the quark–meson model, which naturally encapsulate the information on the spectral functions of mesons. Furthermore, using an approximate

* Corresponding author.
E-mail addresses: sedrakian@fias.uni-frankfurt.de (A. Sedrakian), tripolt@ectstar.eu (R.-A. Tripolt), jwambach@ectstar.eu (J. Wambach).

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form of the input spectral functions of mesons we solve the obtained Eliashberg-type equations exactly, thus fully exhibiting the complex nature of the pairing gap.

After this work was completed, Ref. [13] appeared which studies pairing in the Yukawa model with a finite-range interaction and obtains the full energy-momentum dependence of the gap in the case of imbalanced fermions. It shows that the frequency dependence of the gap in the color-flavor-locked phase of QCD has important ramifications for its color neutrality.

This paper is organized as follows. In Sec. 2 we set up the formalism for 2SC pairing with the quark–meson model and obtain the relevant equations for the pairing gap. Section 3 describes the results of numerical solutions of the gap equations. Our results are summarized in Sec. 4.

2. Formalism

In this work we apply the Nambu–Gorkov formalism where the quark states are combined in spinors (our notations follow Ref. [14])

$$\Psi \equiv \begin{pmatrix} \psi \\ \bar{\psi}^T \end{pmatrix} \equiv \begin{pmatrix} \psi \\ \psi_c \end{pmatrix}.$$

The inverse quark propagator, defined in a standard fashion via the Nambu–Gorkov spinors Ψ, is given by

$$S^{-1}(q) = \begin{pmatrix} \not{q} + \mu\gamma_0 - m & \bar{\Delta} \\ \Delta & (\not{q} - \mu\gamma_0 + m)^T \end{pmatrix}, \qquad (1)$$

where the following relation holds $\bar{\Delta} = \gamma_0 \Delta^\dagger \gamma_0$. We consider the case of equal number densities of up and down quarks with a common chemical potential μ and mass m. The real time-structure of the propagators and self-energies are not specified for simplicity until later. Furthermore, the vertex corrections to the quark–meson vertices $\Gamma^i_\pi(q)$ and $\Gamma_\sigma(q)$ will be neglected and these will be approximated by their bare values

$$\Gamma^i_\pi(q) = \begin{pmatrix} \frac{\tau^i}{2}\gamma_5 & 0 \\ 0 & -(\frac{\tau^i}{2}\gamma_5)^T \end{pmatrix}, \quad \Gamma_\sigma(q) = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}, \qquad (2)$$

where pions are assumed to couple via pseudo-scalar coupling and $\mathbb{I}$ is a unit matrix in the Dirac and isospin spaces. The pion and sigma propagators are given by

$$D_\pi(q) = \frac{1}{q_0^2 - \boldsymbol{q}^2 - m_\pi^2}, \qquad D_\sigma(q) = \frac{1}{q_0^2 - \boldsymbol{q}^2 - m_\sigma^2}, \qquad (3)$$

where $m_{\pi/\sigma}$ are their masses. The gap equation for Δ in the Fock approximation is then given by

$$\Delta(k) = i g_\pi^2 \int \frac{d^4q}{(2\pi)^4} \left(-\frac{\tau^i}{2}\gamma_5\right)^T S_{21}(q) \frac{\tau^j}{2}\gamma_5 \delta_{ij} D_\pi + i g_\sigma^2 \int \frac{d^4q}{(2\pi)^4} (-\mathbb{I})^T S_{21}(q) \mathbb{I} D_\sigma(q-k), \qquad (4)$$

where g_π and g_σ are the coupling constants. The Ansatz for the gap in a 2SC superconductor is given by [3]

$$\Delta^{ab}_{ij}(k) = (\lambda_2)^{ab} (\tau_2)_{ij} C\gamma_5 [\Delta_+(k)\Lambda^+(k) + \Delta_-(k)\Lambda^-(k)], \qquad (5)$$

where $a, b \dots$ refer to the color space, $i, j, \dots$ refer to the flavor space and the projectors onto the positive and negative states are defined as $\Lambda^\pm(k) = (E_k^\pm + \boldsymbol{\alpha}\cdot\boldsymbol{k} + m\gamma_0)/2E_k^\pm$, where $E_k^\pm = \pm\sqrt{\boldsymbol{k}^2 + m^2}$ and $\boldsymbol{\alpha} = \gamma_0 \boldsymbol{\gamma}$. Inverting Eq. (1) one finds for the off-diagonal 21 component of the quark propagator

$$S_{21}(q) = -(\lambda_2\tau_2 C\gamma_5)\left[\frac{\Delta_+\Lambda_-(q)}{q_0^2 - (\epsilon_q - \mu)^2 - \Delta_+^2} + \frac{\Delta_-\Lambda_+(q)}{q_0^2 - (\epsilon_q + \mu)^2 - \Delta_-^2}\right] = -(\lambda_2\tau_2 C\gamma_5) F_{21}(q). \qquad (6)$$

On substituting Eqs. (5) and (6) into Eq. (4) and canceling common terms we find

$$\Delta_+(k)\Lambda^+(k) + \Delta_-(k)\Lambda^-(k) = -i g_\pi^2 \frac{3}{4} \int \frac{d^4q}{(2\pi)^4} \gamma_5 F_{21}(q) \gamma_5 D_\pi(q-k) + i g_\sigma^2 \int \frac{d^4q}{(2\pi)^4} F_{21}(q) D_\sigma(q-k). \qquad (7)$$

In the next step we decompose the remainder of the anomalous propagator into a sum of positive and negative state contributions $F_{21} = \Lambda^- f_1 + \Lambda^+ f_2$. Now, on multiply (7) from the right by $\Lambda^+(k)$ and $\Lambda^-(k)$, using the properties $(\Lambda^\pm)^2 = \Lambda^\pm$, $\Lambda^+ + \Lambda^- = 1$, $\Lambda^+\Lambda^- = 0$, and taking the trace of the resulting two equations (note that Tr $\Lambda^\pm = 4$) we obtain two gap equations introduced in Eq. (5)

$$\Delta_+(k) = -\frac{3 i g_\pi^2}{4} \int \frac{d^4q}{(2\pi)^4} (K_{-+} f_1 + K_{++} f_2) D_\pi(q-k) + \frac{i g_\sigma^2}{4} \int \frac{d^4q}{(2\pi)^4} (M_{-+} f_1 + M_{++} f_2) D_\sigma(q-k). \qquad (8)$$

$$\Delta_-(k) = -i\frac{3 g_\pi^2}{4} \int \frac{d^4q}{(2\pi)^4} (K_{--} f_1 + K_{+-} f_2) D_\pi(q-k) + i\frac{g_\sigma^2}{4} \int \frac{d^4q}{(2\pi)^4} (M_{--} f_1 + M_{+-} f_2) D_\sigma(q-k) \qquad (9)$$

where $K_{\pm\pm} = \mathrm{Tr}[\gamma_5 \Lambda^\pm(q) \gamma_5 \Lambda^\pm(k)]$ and $M_{\pm\pm} = \mathrm{Tr}[\Lambda^\pm(q)\Lambda^\pm(k)]$. The commutation property $[\Lambda^\pm, \gamma^5] = 0$ implies that we may set in Eqs. (8) and (9) $K_{\pm\pm} = M_{\pm\pm}$. A further simplification arises because one is generally interested in the gap at the Fermi surface of the particles and it is legitimate to drop the antiparticle component of the decomposition of the gap function (5) and take $\Delta_- = 0$. Indeed the integrand of Eq. (8) is strongly peaked at the Fermi surface, i.e., when $\epsilon_q = \mu$ due to the pole structure of the anomalous propagator (6). Its antiparticle pole is located at energies $2\mu \sim 700$ MeV and, therefore, cannot influence the physics at much lower scale $\sim \Delta_+ \ll 2\mu$.

We find then

$$\Delta_+(k) = -i\frac{3 g_\pi^2}{4} \int \frac{d^4q}{(2\pi)^4} K_{-+} f_1(k-q) D_\pi(q) + i\frac{g_\sigma^2}{4} \int \frac{d^4q}{(2\pi)^4} K_{-+} f_1(k-q) D_\sigma(q), \qquad (10)$$

where $f_1 = \Delta_+/(q_0^2 - \xi_q^2 - \Delta_+^2)$ with $\xi_q = \varepsilon_q - \mu$. At this point we make explicit the finite-temperature content of the equations above within the Schwinger–Keldysh real-time formalism. The retarded component of the gap function can be written in standard notations [15,16]

$$\Delta_+^R(k_0) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \times \frac{D^>(\omega')F^>(\omega-\omega') - D^<(\omega')F^<(\omega-\omega')}{k_0 - \omega + i\delta}, \qquad (11)$$

where