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Higher-n triangular dilatonic black holes

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ABSTRACT

Dilaton gravity with the form fields is known to possess dyon solutions with two horizons for the discrete "triangular" values of the dilaton coupling constant $a=\sqrt{n(n+1)/2}$. This sequence first obtained numerically and then explained analytically as consequence of the regularity of the dilaton, should have some higher-dimensional and/or group theoretical origin. Meanwhile, this origin was explained earlier only for $n=1,2$ in which cases the solutions were known analytically. We extend this explanation to $n=3,5$ presenting analytical triangular solutions for the theory with different dilaton couplings a, b in electric and magnetic sectors in which case the quantization condition reads $ab=n(n+1)/2$. The solutions are derived via the Toda chains for B_2 and G_2 Lie algebras. They are found in the closed form in general D space–time dimensions. Solutions satisfy the entropy product rules indicating on the microscopic origin of their entropy and have negative binding energy in the extremal case.

1. Introduction

Einstein–Maxwell-dilaton (EMD) theory in four dimensions may originate from different supersymmetric higher-dimensional theories with various values of the dilaton coupling constant a in the Maxwell term $e^{-2a\phi}F^2$ in the Lagrangian. For $a=0$ the dilaton decouples and the theory reduces to Einstein–Maxwell (EM) system, which is the bosonic part of $N=2$, $D=4$ supergravity. The value $a=1$ corresponds to $N=4$, $D=4$ supergravity [1,2]. The value $a=\sqrt{3}$, corresponds to dimensionally reduced $D=5$ gravity [3,4], which have different supersymmetric extensions. In all these cases analytical solutions are known for static black holes possessing both electric and magnetic charges (dyons) with two horizons between which the dilaton exhibits n oscillations. Such solutions have non-singular extremal limits with $AdS_2\times S^2$ horizons contrary the one-charged dilatonic black hole which have singular horizons in the extremal limit [1,2,5] for generic coupling constant.

As it was first noticed by Poletti, Twamley and Wiltshire [6], the values $a=1,\sqrt{3}$ are just the two lowest members $n=1,2$ of the "triangular" sequence of dilaton couplings

$$a_n=\sqrt{n(n+1)/2}, \tag{1}$$

for which dyons (known for higher n only numerically) exhibit similar behavior of the dilaton. Analytically this triangle quantization rule was rederived in [7] as condition of regularity of the dilaton in the case of coinciding horizons, i.e. for extremal dyons. The same rule was shown to arise from the linearized dilaton equation on the Reissner–Nordström dyonic background [8,9] as condition of existence of dilaton bound states. However, exact higher-n dyonic solutions were not known analytically. Here we give solutions for $n=3,5$ in the theory where the dilaton couples to electric and magnetic sectors with different coupling constants a, b, in which case the quantization rule generalizes to $ab=n(n+1)/2$. We also give generalization to arbitrary dimensions. These new solutions are derived via Toda chains for B_2 and G_2 algebras. The solutions satisfy the entropy product rule: the product of the entropies of the internal and internal horizons are functions of their charges only [10,11] (see also [8]). This property is considered to be an indication on possibility of statistical interpretation of the entropy. Another interesting feature is that the extremal dyons have negative binding energy like the $SL(n,R)$ multi-scalar Toda black holes [12].

Previous work on Toda dilatonic black holes includes among other [13–16]. Our results have some overlap with the recent preprints [9,17].

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2. Toda black holes

2.1. Setup

Consider Einstein–Maxwell-dilaton system in D dimensions

$$S=\int d^Dx\sqrt{-g}\times\left(R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2(D-2)!}e^{a\phi}F_{[D-2]}^2-\frac{1}{4}e^{b\phi}F_{[2]}^2\right), \quad (2)$$

with two different dilaton coupling constants a, b for the magnetic $(D-2)$-form $F_{[D-2]}$ and the electric 2-form $F_{[2]}$. Assume the static spherically symmetric ansatz for the metric

$$ds^2=-e^{2B}dt^2+e^{2A}dr^2+e^{2C}d\Omega_{D-2}^2, \quad (3)$$

and the following solution of the equations of motion for the form fields:

$$F_{[D-2]}=P\,\mathrm{vol}(\Sigma_{D-2}), \qquad F^{rt}=Q\,e^{-b\phi}e^{-A-B-(D-2)C}. \quad (4)$$

All the unknown functions A, B, C, ϕ depend on a single variable r. One convenient gauge choice is $A(r)=-B(r)$. Denoting then $e^{C(r)}=R(r)$, from (2) one can derive the following four equations for three functions B, R, ϕ (one of which following from the other three by the Bianchi identity):

$$\begin{aligned}&\left(R^{D-2}(e^{2B})'\right)'=\frac{D-3}{(D-2)R^{D-2}}\left(P^2e^{a\phi}+Q^2e^{-b\phi}\right),\\&\left(e^{2B}(R^{D-2})'\right)'\\&\quad=-\frac{1}{2R^{D-2}}\left(P^2e^{a\phi}+Q^2e^{-b\phi}\right)+(D-2)(D-3)R^{D-4},\\&R''+\frac{1}{2(D-2)}R\phi'^2=0,\\&\phi''+\phi'\left(e^{2B}R^{D-2}\right)'e^{-2B}R^{-(D-2)}\\&\quad=\frac{1}{2}e^{-2B}\left(aP^2e^{a\phi}-bQ^2e^{-b\phi}\right)R^{-2(D-2)}. \quad (5)\end{aligned}$$

2.2. Triangular quantization

The dilaton coupling quantization can be obtained analytically for extremal dyons as condition of regularity of the dilaton at the degenerate event horizon $r=r_0$, where the expansion of e^{2B} in $x\equiv(r^{D-3}-r_0^{D-3})/r_0^{D-3}$ starts with x^2:

$$\begin{aligned}&R=R_h+\rho_1x+\rho_2x^2+O(x^3),\\&e^{2B}=\nu_2x^2+O(x^3),\\&\phi=\phi_h+\mu_nx^n+O(x^{n+1}), \quad n\in\mathbb{Z}. \quad (6)\end{aligned}$$

The leading power index n of the dilaton is assumed to be integer to ensure analyticity of the solution. The second and the third equations in (5) yield the following expressions for R_h and ρ_1, ρ_2

$$\begin{aligned}&R_h^{2(D-3)}=\frac{P^2e^{a\phi_h}+Q^2e^{-b\phi_h}}{2(D-3)(D-2)},\\&\frac{D-4}{2(D-3)}\rho_1+\rho_2=-\frac{R_h\mu_n^2}{4(D-2)}\delta_{n,1}. \quad (7)\end{aligned}$$

The other equations lead to the following relations

$$\begin{aligned}&aP^2e^{a\phi_h}-bQ^2e^{-b\phi_h}=0,\\&2\nu_2n(n+1)=\frac{r_0^2}{(D-3)^2R_h^{2(D-2)}}\left(a^2P^2e^{a\phi_h}+b^2Q^2e^{-b\phi_h}\right),\\&2\nu_2=\frac{r_0^2}{(D-2)(D-3)R_h^{2(D-2)}}\left(P^2e^{a\phi_h}+Q^2e^{-b\phi_h}\right). \quad (8)\end{aligned}$$

The first equation in (8) states that the value ϕ_h is completely determined by charges and coupling constants. Using it in the first equation of (7) and assuming that the solution can be continued to infinity as a global black hole, we find for its Bekenstein entropy $S=A_h/4$ the following expression

$$\begin{aligned}S_{\rm ext}&=\frac{\pi^{\frac{D-1}{2}}}{2\Gamma\left(\frac{D-1}{2}\right)}R_h^2\\&=\frac{\pi^{\frac{D-1}{2}}}{2\Gamma\left(\frac{D-1}{2}\right)}\left[\left(\frac{a}{b}\right)^{-\frac{a}{a+b}}\frac{a+b}{2b(D-2)(D-3)}P^{\frac{2b}{a+b}}Q^{\frac{2a}{a+b}}\right]^{\frac{1}{D-3}}, \quad (9)\end{aligned}$$

depending solely on the charges.

Using the first equation in (8) to simplify the ratio of the second and the third ones, one finally obtains the necessary condition for the product of coupling constants:

$$ab=n(n+1)\frac{D-3}{D-2}. \quad (10)$$

Redefining them as

$$(a,b)=\lambda(\tilde a,\tilde b), \qquad \lambda^2=\frac{2(D-3)}{D-2}, \quad (11)$$

we get

$$\tilde a\tilde b=\frac{n(n+1)}{2}, \quad (12)$$

which for $a=b$ coincides with (1).

It is worth mentioning that the theory in $D=5$ with exactly the same lagrangian (2) was discussed in the literature in quite a different context (black rings) by Yazadjiev [18]. Analytical solutions for dyons and black rings obtained there correspond to our triangular sequence with $n=1$, which was mentioned in the Introduction as originating from $D=4$, $N=4$ supergravity. It follows that this case can have another five-dimensional interpretation as well, thus manifesting duality symmetries of dimensionally reduced theories. From the Toda point of view, this case corresponds to the $A_1\times A_1$ Lie algebra.

2.3. Toda representation

Passing to another gauge[1]:

$$e^{2C}=e^{2A}fr^2, \qquad f=1-\frac{2\mu}{r^{D-3}}, \quad (13)$$

one obtains $e^{2A}=e^{-\frac{2B}{D-3}}f^{-\frac{D-4}{D-3}}$, so the metric reduces to

$$ds^2=-e^{2B}dt^2+e^{-\frac{2B}{D-3}}f^{\frac{1}{D-3}}\left(f^{-1}dr^2+r^2d\Omega_{D-2}^2\right). \quad (14)$$

The function f, often called the blackening factor, has one zero corresponding to the black-hole horizon and located at $r=$

[1] In this gauge radial coordinate r is different from that in the previous subsections. We still use the same symbol r to simplify notation.