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Constraining the loop quantum gravity parameter space from phenomenology

Suddhasattwa Brahma [a,b], Michele Ronco [c,d,*]

[a] Center for Field Theory and Particle Physics, Fudan University, Shanghai 200433, China
[b] Asia Pacific Center for Theoretical Physics, Pohang 37673, Republic of Korea
[c] Dipartimento di Fisica, Università di Roma "La Sapienza", P.le A. Moro 2, 00185 Roma, Italy
[d] INFN, Sez. Roma1, P.le A. Moro 2, 00185 Roma, Italy

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ABSTRACT

Development of quantum gravity theories rarely takes inputs from experimental physics. In this letter, we take a small step towards correcting this by establishing a paradigm for incorporating putative quantum corrections, arising from canonical quantum gravity (QG) theories, in deriving *falsifiable* modified dispersion relations (MDRs) for particles on a deformed Minkowski space–time. This allows us to differentiate and, hopefully, pick between several quantization choices via *testable, state-of-the-art* phenomenological predictions. Although a few explicit examples from loop quantum gravity (LQG) (such as the regularization scheme used or the representation of the gauge group) are shown here to establish the claim, our framework is more general and is capable of addressing other quantization ambiguities within LQG and also those arising from other similar QG approaches.

1. Introduction and motivation

It is well-known that the lack of experimental evidence represents one of the main obstacles in our search for a theory of QG [1–4]. In the absence of observations, researchers often rely on less dependable principles, such as 'beauty' and 'naturalness', as guidance for advancing QG proposals [5]. Working within a given approach, one is then usually forced to choose between quantization ambiguities, often on the same footing theoretically, by following one's personal penchants or other questionable criteria. In this letter, we relate different quantization schemes, which have been proposed in the LQG literature [6–9], to different predictions for observable quantities. And by doing so, we lay down a framework to distinguish between them using observations.

LQG is a non-perturbative, background-independent approach to quantize gravity [10,11], with significant accomplishments such as 'singularity resolution' in various cosmological and black-hole scenarios [12,13]. However, as in other QG models, conclusions typically depend on various quantization choices. So far very little work has been directed towards understanding whether these formal alternatives affect physical outcomes. Among other reasons, this is largely a consequence of the fact that the complexity of the full-fledged theory has created a gap between technical results and potential observations.

Remarkably, recent results in symmetry-reduced LQG models which, in particular, has focussed on the study of quantum symmetries in the presence of LQG-inspired corrections [14–18], have unanimously discovered the fact that general covariance should be *modified* by such quantum effects. These modifications amount to a deformation of the brackets closed by the gravitational constraints which generate space and time gauge transformations. Here, we outline a path to derive MDRs corresponding to the modified brackets and show that quantization ambiguities leave their imprints on the form of the MDR. This would suggest that different quantization schemes adopted (and often treated interchangeably) are not equivalent and, conceivably, might be distinguished thanks to forthcoming tests of Planck-scale departures from special relativistic symmetries [4,19–22].

Although we focus on particular quantization choices characteristic to LQG (such as the choice of the Immirzi parameter, the regularization scheme used or the dimension of the gauge group), we shall unequivocally demonstrate that our analysis is general enough to include other such ambiguities in LQG as well as for corrections coming from other canonical QG approaches.

* Corresponding author.
E-mail addresses: suddhasattwa.brahma@gmail.com (S. Brahma), michele.ronco@roma1.infn.it (M. Ronco).

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Throughout the paper we work with natural units if not specified differently.

2. Deformed covariance and modified dispersion relations

One of the newest results in LQG is the emergence of non-classical space–times structures [23–31]. These departures from smooth classical space–time manifolds can be meaningfully traced back to quantum modifications of the so-called hypersurface deformation algebra (HDA), which encodes covariance in the Hamiltonian formulation of classical general relativity [32]. In fact, since the structure function appearing in the classical HDA is the inverse of the spatial metric h^{ab} on the hypersurface (see e.g. [33] as well as the equation below), then it is believed that any modification (usually by a phase-space function, β) to it points towards a deformation of the space–time geometry. The common feature of LQG models is that only the bracket between two generators of normal deformations (or the Hamiltonian constraint) is affected:

$$\{H^Q[N], H^Q[M]\} = D[\beta h^{ab}(N\partial_b M - M\partial_b N)]\,. \tag{1}$$

Gauge transformations generated by constraints (H[1] and D for normal N lapse and tangential N^a shifts respectively) represent coordinate freedom in classical canonical gravity. The closure of the brackets assures there is no violation of the gauge symmetries, but the modification in the above equation implies a *deformed* notion of covariance [17,18,30].

It is known that the Poincaré algebra, which describes symmetries of Minkowski space–time, can be derived as a special case from the classical HDA in a systematic manner [33,34] (see below for a short review of the procedure). Not surprisingly, LQG-deformations of the HDA turn into corresponding deformations of the Poincaré algebra [35–37]. As a consequence, as first shown in [38], the familiar dispersion relation (for massless particles), $E^2 = p^2$, does not hold true anymore and is replaced by more complicated expressions.

Usually, due to the complexity of QG theories, MDRs are either parametrized as a generic series expansion in (inverse) powers of $m_{\rm Pl}$ with some unknown coefficients, i.e. $E^2 \simeq p^2 + a_1 p^3/m_{\rm Pl} + a_2 p^4/m_{\rm Pl}^2$ (where $a_1, a_2, \ldots, a_n$ are to be determined experimentally) as a purely phenomenological ansatz, or derived in simplified models (see e.g. [4,39–42]). Taking the opposite direction here, we compute MDRs from a fundamental QG theory – LQG – and, thus, contribute to bridge the gap between top-down and bottom-up approaches. From this perspective, our work is also part of an on-going effort [37,43] aimed at characterizing the Minkowski limit of LQG, and exploring if there is any relation to non-commutative geometries [44–47] as a way to characterize the so-called *spacetime fuzziness* or *foaminess* [48–51].

2.1. Deriving MDRs from deformed-HDA

In this section, we take the Minkowski limit of the LQG-deformed HDA and eventually prove that it affects the dispersion relation through a corresponding deformation of the Poincaré algebra. The full deformed-HDA is given by

$$\begin{aligned} \{D[M^a], D[N^a]\} &= D[\mathcal{L}_{\vec{M}} N^a], \\ \{D[N^a], H^Q[M]\} &= H^Q[\mathcal{L}_{\vec{N}} M], \\ \{H^Q[M], H^Q[N]\} &= D[\beta h^{ab}(M\partial_b N - N\partial_b M)]\,. \end{aligned} \tag{2}$$

[1] The superscript 'Q' implies that we are dealing with the LQG quantum-corrected Hamiltonian constraint here.

In order to reduce to the flat limit, one has to restrict to linear lapse and shift functions, which correspond to linear coordinate changes, i.e.

$$N^k(x) = \Delta x^k + R^k_i x^i \quad N(x) = \Delta t + v_i x^i \tag{3}$$

and, at the same time, to flat spatial hypersurfaces i.e. $h_{ij} \equiv \delta_{ij}$. With these restrictions one can prove that the infinite set of general diffeomorphisms reduce to the finite subset of Poincaré transformations. It is then possible to read off the commutators between the Poincaré generators directly from the HDA.In particular, from $\{D[M^a], D[N^a]\}$ one can derive $\{J_i, J_j\}, \{J_i, P_j\}$, and $\{P_i, P_j\}$ (J_i being the generator of rotations and P_i that of spatial translations), while $\{J_i, N_j\}, \{P_0, J_j\}, \{N_i, P_j\}$ and $\{P_i, P_0\}$ (N_i being the generator of boosts and P_0 that of time translations) can be obtained from $\{D[N^a], H^Q[M]\}$, and finally from $\{H^Q[M], H^Q[N]\}$ one gets $\{N_i, N_j\}$ and $\{N_i, P_0\}$. In the appendix, we explicitly illustrate the case of rotations as an example.

Let us start with the spherically-symmetric reduction of Hamiltonian gravity in Ashtekar–Barbero variables (see e.g. [52]) in the presence of LQG deformations. In this case the ADM foliation [32] allows to decompose the space–time manifold as $\mathcal{M} = \mathbb{R} \times \Sigma = \mathcal{M}_{1+1} \times S^2$, where $\mathcal{M}_{1+1}$ is a 2-dimensional manifold spanned by (t, r) and S^2 stands for the 2-sphere. Given that, the line element reads

$$ds^2 = -N^2 dt^2 + h_{rr}(dr + N^r dt)^2 + h_{\theta\theta}(d\theta^2 + \sin^2\theta\varphi^2)\,, \tag{4}$$

where the shift vector is purely radial, *i.e.* $N^i = (N^r, 0, 0)$, due to spherical symmetry, and, consequently, we are left only with radial diffeomorphisms generated by $D[N^r] = \int dr N^r \mathcal{H}_r$ (where $\mathcal{H}_r$ is the only non-vanishing component of the momentum density) and, time transformations, generated by $H[N] = \int dr N \mathcal{H}$ (where $\mathcal{H}$ is the Hamiltonian density). The components of the spatial metric $(h_{rr}, h_{\theta\theta})$ can be written in terms of rotationally invariant densitized triads which are given by:

$$\begin{aligned} E = E^a_i \tau^i \frac{\partial}{\partial x^a} &= E^r(r)\tau_3 \sin\theta \frac{\partial}{\partial r} + \\ &+ E^\varphi(r)\tau_1 \sin\theta \frac{\partial}{\partial\theta} + E^\varphi(r)\tau_2 \frac{\partial}{\partial\varphi}\,, \end{aligned} \tag{5}$$

where $\tau_j = -\frac{1}{2} i\sigma_j$ represent $SU(2)$ generators. The densitized triads are canonically conjugate to the extrinsic curvature components, which, in presence of spherical symmetry, are conveniently described as follows

$$\begin{aligned} K = K^i_a \tau_i dx^a = K_r(r)\tau_3 dr + K_\varphi(r)\tau_1 d\theta + \\ + K_\varphi(r)\tau_2 \sin\theta d\varphi\,. \end{aligned} \tag{6}$$

For the simplest case including only local holonomy corrections [53,54], with $\gamma \in \mathbb{R}$ and $j = 1/2$, the deformation β takes the form

$$\beta = \cos(2\delta K_\varphi)\,, \tag{7}$$

where δ is a regularization parameter, related to the square root of the minimum eigenvalue of the area operator.

As already discussed in [37], the main difficulty lies in the fact that LQG-deformations in the HDA arises in the form of the structure function getting modified by a function of the phase space variables, while deformations at the level of the Poincaré algebra usually implies modification of the algebra generators [41,42]. As a way out, it is then convenient to find a way to write β in terms of symmetry generators (see also [35,36,38]), and for this purpose, it