Physics Letters B 779 (2018) 275–282

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Gauge copies in the Landau–DeWitt gauge: A background invariant restriction

David Dudal [a,b,*], David Vercauteren [c]

[a] KU Leuven Campus Kortrijk – Kulak, Department of Physics, Etienne Sabbelaan 53 bus 7657, 8500 Kortrijk, Belgium
[b] Ghent University, Department of Physics and Astronomy, Krijgslaan 281-S9, 9000 Gent, Belgium
[c] Duy Tân University, Institute of Research and Development, P809, 3 Quang Trung, Hải Châu, Đà Nẵng, Viet Nam

ARTICLE INFO

Article history:
Received 1 December 2017
Received in revised form 5 February 2018
Accepted 9 February 2018
Available online 13 February 2018
Editor: M. Cvetič

ABSTRACT

The Landau background gauge, also known as the Landau–DeWitt gauge, has found renewed interest during the past decade given its usefulness in accessing the confinement-deconfinement transition via the vacuum expectation value of the Polyakov loop, describable via an appropriate background. In this Letter, we revisit this gauge from the viewpoint of it displaying gauge (Gribov) copies. We generalize the Gribov–Zwanziger effective action in a BRST and background invariant way; this action leads to a restriction on the allowed gauge fluctuations, thereby eliminating the infinitesimal background gauge copies. The explicit background invariance of our action is in contrast with earlier attempts to write down and use an effective Gribov–Zwanziger action. It allows to address certain subtleties arising in these earlier works, such as a spontaneous and thus spurious Lorentz symmetry breaking, something which is now averted.

1. Introduction

A powerful quantization procedure for locally gauge invariant Yang–Mills theories is the background field formalism, in which formalism the gauge field is split in a non-propagating "classical" background and a fluctuating quantum part which is integrated over in the path integral procedure. Just as when dealing with an ordinary gauge theory, the quantum gauge fields need to be gauge fixed in the continuum. A particularly useful class of gauges in this context are the background covariant gauges; in these gauges, the background field formalism possesses the important property that, after gauge fixing of and integration over the quantum fields, the eventual (effective) action ought to still be invariant with respect to gauge transformations of the background fields. Useful references are [1,2].

Background gauges found a renewed interest during the past decade thanks to their usefulness in probing a typical (non-local) order parameter for the deconfinement transition, the Polyakov loop, whose behaviour can be encoded in a simple specific background, see [3–9]. Also in the pinch technique combined with Dyson–Schwinger equations, the background field formalism plays a central role [10–12]. Algebraic aspects of a specific background gauge, the Landau–DeWitt one, including an all order renormalizability proof, were considered in [13–15].

Albeit powerful, the (covariant) background gauges are also not free from the famous Gribov ambiguity [16] hampering the quantization: multiple gauge equivalent copies of a given quantum gauge field obey the same gauge condition. To deal with this ambiguity, one possibility is to further constrain the space of gauge configurations to be integrated over in the path integral, a procedure proposed by Gribov in [16] and worked out by Zwanziger in e.g. [17,18] for the standard Landau gauge. The end point is an effective action — the Gribov–Zwanziger action — implementing this restriction. More references can be found in [19].

In the presence of backgrounds, seminal work is [20], based on which a background version of the Gribov–Zwanziger effective action was proposed and used to probe non-perturbative finite temperature dynamics in [21,22].

In this Letter, we revisit in Section 2 the problem of Gribov copies in the Landau–DeWitt gauge and try to *derive* a Gribov–Zwanziger action. Although succeeding in the latter, we identify a major drawback, shared with the *conjectured* action in [21,22]: even at zero temperature, a non-zero value of a Lorentz symmetry breaking background is energetically favoured. The problem is traced back to the lack of background gauge invariance and, underlyingly, of BRST invariance of the original Gribov–Zwanziger approach. Motivated by the observation in [13] that in the back-

* Corresponding author.
E-mail addresses: david.dudal@kuleuven.be (D. Dudal), vercauterendavid@dtu.edu.vn (D. Vercauteren).

https://doi.org/10.1016/j.physletb.2018.02.014
0370-2693/

ground field formalism, BRST invariance at the quantum level is closely linked to background gauge invariance at the classical level, in Section 3 we then go on remedying this problem by constructing a BRST and background invariant version of the Gribov–Zwanziger action, the latter still capable of mitigating the Gribov copy problem but no longer exhibiting the undesirable unphysical features at zero temperature.

2. Gribov–Zwanziger with a background

Let us initially work in a general SU(N) gauge theory, such that the structure constants will be written as f^{abc}. Later we will restrict to SU(2) and choose a particular form for the background field.

Our objective is to compute the path integral of Yang–Mills theory in perturbation theory around a given background $\bar{A}^a_\mu$. We therefore split the total gluon field a^a_μ as $\bar{A}^a_\mu + A^a_\mu$, where A^a_μ are the quantum fluctuations around the classical background field $\bar{A}^a_\mu$. Instead of the usual Landau gauge $\partial_\mu a^a_\mu = 0$ we will choose the Landau background gauge or Landau–DeWitt gauge $\bar{\mathcal{D}}^{ab}_\mu A^b_\mu = 0$, where $\bar{\mathcal{D}}^{ab}_\mu = \delta^{ab}\partial_\mu - g f^{abc}\bar{A}^c_\mu$ is the covariant derivative containing the background field.

We will now give a first way to adapt the Gribov–Zwanziger framework [16–18] to the case with a background, naively following the same steps as led to the original framework without background.

In case of the (transverse) Landau gauge, $\partial_\mu A^a_\mu = 0$, the Gribov–Zwanziger action arises from the restriction of the domain of integration in the Euclidean functional integral to the so-called Gribov region Ω, which is defined as the set of all gauge field configurations fulfilling the gauge $\partial_\mu A^a_\mu = 0$ and for which the (Hermitian) Faddeev–Popov operator is strictly positive. Indeed, requiring positivity of the Faddeev–Popov operator excludes infinitesimal gauge copies, as such copies are connected as $A^a_\mu \to A^a_\mu + \mathcal{D}^{ab}_\mu \omega^b$ and can both be transverse whenever $-\partial_\mu \mathcal{D}^{ab}_\mu \omega^b = 0$.

Generalizing to the case at hand, this means that we should restrict to

$$\Omega = \{A^a_\mu \mid \bar{\mathcal{D}}^{ab}_\mu A^b_\mu = 0 \;\&\; \mathcal{M}^{ac} = -\bar{\mathcal{D}}^{ab}_\mu(\bar{\mathcal{D}}^{bc}_\mu - g f^{bcd}A^d_\mu) > 0\}\,. \quad (1)$$

The starting point is the (Euclidean) Faddeev–Popov action in the chosen gauge:

$$S_{\text{FP}} = \int d^4x \left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + b^a \bar{\mathcal{D}}^{ab}_\mu A^b_\mu + \bar{c}^a \bar{\mathcal{D}}^{ab}_\mu(\bar{\mathcal{D}}^{bc}_\mu - g f^{bcd}A^d_\mu)c^c\right), \quad (2)$$

where $(\bar{c}^a, c^a)$ stand for the Faddeev–Popov ghosts, b^a is the Lagrange multiplier implementing the Landau gauge, and $F^a_{\mu\nu}$ denotes the field strength containing the full gluon field $a^a_\mu = \bar{A}^a_\mu + A^a_\mu$:

$$F^a_{\mu\nu} = \partial_\mu a^a_\nu - \partial_\nu a^a_\mu + g f^{abc} a^b_\mu a^c_\nu\,. \quad (3)$$

As is generally known, this formalism restricts the path integral to those gluon field configurations obeying the gauge condition. However, this construction still includes many configurations for which the Faddeev–Popov operator is not strictly positive.

In order to impose the second condition, we will construct the no-pole condition [16]. Following [23] we can compute this condition to all orders instead of expanding in the gluon field. We aim to invert the Faddeev–Popov operator $\mathcal{M}^{ac} = -\bar{\mathcal{D}}^{ab}_\mu(\bar{\mathcal{D}}^{bc}_\mu - g f^{bcd}A^d_\mu)$. Let us introduce the operator σ^{ab} — which depends on $\bar{A}^a_\mu$ and on A^a_μ — as

$$(\mathcal{M}^{-1})^{ae} = \left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{ab}\left(\delta^{be} - g\bar{\mathcal{D}}^{bc}_\mu f^{cdf}A^f_\mu\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{de} + \sigma^{be}\right). \quad (4)$$

The condition of having a strictly positive Faddeev–Popov operator is equivalent to requiring that the sum of all connected diagrams contributing to the ghost propagator be always finite [23], or that

$$\mathcal{G}(k) = \langle \mathcal{G}(k,A)\rangle^{\text{conn}} = \langle \mathcal{M}^{-1}\rangle^{\text{conn}} < \infty\,, \forall k\,. \quad (5)$$

Using the expression (4) and taking account of the facts that the background covariant derivatives $\bar{\mathcal{D}}_\mu$ do not interact with the taking of the vacuum expectation value (meaning the derivatives can be put in front of the brackets) and that the quantum fields have vanishing vacuum expectation value $\langle A^a_\mu\rangle = 0$, we find that the condition for a strictly positive Faddeev–Popov operator can be written as

$$\mathcal{G}^{ac}(k) = \left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{ab}\langle \delta^{bc} + \sigma^{bc}\rangle^{\text{conn}} = \left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{ab}\left(\frac{1}{1 - \langle\sigma\rangle^{1\text{PI}}}\right)^{bc} < \infty\,, \forall k\,. \quad (6)$$

In our case, the operator $-\bar{\mathcal{D}}^2$ will be positive definite (apart from the usual trivial zero), such that the condition we are seeking is equivalent to the no-pole condition that the eigenvalues of $\langle\sigma\rangle^{1\text{PI}}$ be less than one.

As usual, we will introduce the no-pole condition into the path integral by using the Fourier representation of the Heaviside function:

$$\int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{d\beta}{2\pi i\beta} e^{\beta - \beta\sigma(0)}\,, \quad (7)$$

where the notation $\sigma(0)$ stands for the highest eigenvalues of the σ operator. In the case without background, this means the momentum flowing through σ is simply set to zero. As a result of this way of introducing the no-pole condition, an extra part $-\beta + \beta\sigma(0)$ will be added to the action. The integration variable β will be called the Gribov parameter. The integral over the Gribov parameter is normally done using the steepest descent method, which leads to a gap equation for β.

Let us now solve (4) for σ. After some trivial reordering, we can write that

$$\sigma^{ab} = (-\bar{\mathcal{D}}^2)^{ac}(\mathcal{M}^{-1})^{cb} - \delta^{ab} + g\bar{\mathcal{D}}^{ac}_\mu f^{cdf}A^f_\mu\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{db}. \quad (8)$$

Now replace the Kronecker delta by $-\bar{\mathcal{D}}^2\mathcal{M}\mathcal{M}^{-1}(-\bar{\mathcal{D}}^2)^{-1}$:

$$\begin{aligned}\sigma^{ab} &= (-\bar{\mathcal{D}}^2)^{ac}(\mathcal{M}^{-1})^{cd}\left(\delta^{db} - \mathcal{M}^{de}\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{eb}\right) + g\bar{\mathcal{D}}^{ac}_\mu f^{cdf}A^f_\mu\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{db}\\ &= -g(-\bar{\mathcal{D}}^2)^{ac}(\mathcal{M}^{-1})^{cd}\bar{\mathcal{D}}^{de}_\mu f^{efh}A^h_\mu\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{fb} + g\bar{\mathcal{D}}^{ac}_\mu f^{cdf}A^f_\mu\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{db}\\ &= -g\left((-\bar{\mathcal{D}}^2)^{ac}(\mathcal{M}^{-1})^{cd} - \delta^{ad}\right)\bar{\mathcal{D}}^{de}_\mu f^{efh}A^h_\mu\left(\frac{1}{-\bar{\mathcal{D}}^2}\right)^{fb},\end{aligned} \quad (9)$$