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Stability issues of black hole in non-local gravity

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ABSTRACT

We discuss stability issues of Schwarzschild black hole in non-local gravity. It is shown that the stability analysis of black hole for the unitary and renormalizable non-local gravity with $\gamma_2 = -2\gamma_0$ cannot be performed in the Lichnerowicz operator approach. On the other hand, for the unitary and non-renormalizable case with $\gamma_2 = 0$, the black hole is stable against the metric perturbations. For non-unitary and renormalizable local gravity with $\gamma_2 = -2\gamma_0 = \text{const}$ (fourth-order gravity), the small black holes are unstable against the metric perturbations. This implies that what makes the problem difficult in stability analysis of black hole is the simultaneous requirement of unitarity and renormalizability around the Minkowski spacetime.

1. Introduction

It turns out that the infinite derivative gravity (non-local gravity) is ghost-free and renormalizable around the Minkowski spacetime background when one chooses the exponential form of an entire function [1,2]. We note that renormalizability can be easily checked by showing the finiteness of the Newtonian potential at the origin from the propagator [3–6].

On the other hand, all Ricci-flat spacetimes including Schwarzschild black hole are exact solutions for non-local gravitational theories [7]. In order to check that the Schwarzschild black hole exists really in the unitary (ghost-free) and renormalizable non-local gravity, one has to perform the stability analysis of the black hole. If the black hole solution passes the stability test, one may save that black hole. Recently, it has shown that the Schwarzschild black hole is stable against linear perturbations for a subclass of unitary non-local gravity with $\gamma_2 = 0$ [8]. However, this case is not a renormalizable gravity around Minkowski spacetime. Although the non-locality (operators with infinitely many derivatives) is needed to have a ghost-free and renormalizable gravity, the presence of higher derivative gravity may make the black hole unsustainable. This implies that non-locality may not be a good tool to cure the black hole solutions.

In this work, we wish to discuss stability issues of Schwarzschild black hole in non-local gravity. We derive a linearized equation (21) which governs the stability of black hole for the unitary and renormalizable non-local gravity with $\gamma_2 = -2\gamma_0$. However, we could not perform the stability analysis of black hole in the Lichnerowicz operator approach because the appearance of entire function in the linearized equation (34). On the other hand, for a unitary and non-renormalizable gravity with $\gamma_2 = 0$, it has shown that the black hole is stable against the metric perturbations. This is possible because this case reduces to the Einstein gravity or $f(R)$ gravity [9], which are surely independent of the entire function [8]. Next, for the non-unitary and renormalizable local gravity with $\gamma_2 = -2\gamma_0 = \text{const}$ (fourth-order gravity) [10], using the Gregory–Laflamme black string instability [11], the small black holes are unstable against the Ricci tensor perturbations. This contrasts to the conventional stability analysis of black hole in Einstein gravity or $f(R)$ gravity. It implies that the simultaneous requirement for unitarity and renormalizability makes the stability analysis difficult.

2. Non-local gravity

A non-local gravity in four dimensions is generally defined by [8]

$$S_g = \frac{2}{\kappa^2}\int d^4x\sqrt{|g|}\left[R + R\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + V_g\right], \quad (1)$$

where $\kappa^2 = 32\pi G$, the d'Alembertian $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, and the potential term V_g is at least cubic in the curvature and at least quadratic in the Ricci tensor. Hereafter, we choose $V_g = 0$ for simplicity. The non-local gravity with $\gamma_2 = 0$ is unitary and non-

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renormalizable around Minkowski spacetime when choosing one of the two form factors

$$\gamma_0=-\frac{e^{H(\Box)}-1}{6\Box}, \tag{2}$$

$$\gamma_0=-\frac{e^{H(\Box)}\left(1-\frac{\Box}{M^2}\right)-1}{6\Box}, \tag{3}$$

where $H(\Box)$ is an entire function and M is a mass scale. The first form factor was first proposed by Biswas, Mazumdar and Siegel with $H(\Box)=\Box$ [12], whereas the second one appears in the non-local extension of Starobinsky gravity [13].

For $\gamma_2=-2\gamma_0$, the tree-level unitarity analysis for the metric perturbation around Minkowski spacetime shows that the graviton propagator for (2) takes the form of $\Pi(k)=e^{-H(-k^2)}\Pi_{\rm GR}$ [4]. Furthermore, its renormalizability can be easily seen by computing the Newtonian potential from this propagator.

Before we proceed, we would like to note that the unitary case of $\gamma_2=0$ was reduced to the stability analysis of the Schwarzschild black hole in Einstein gravity for the case of (2) and $f(R)$ gravity for the case of (3) [8]. However, this case is not a renormalizable gravity when quantizing around Minkowski spacetime. Therefore, in order to make a connection to the unitary and renormalizable non-local gravity, one considers the case of $\gamma_2=-2\gamma_0$ in the beginning of stability analysis for the black hole.

3. Equation of motion: Ricci-flat solutions

The equation of motion is derived from the action (1) as [14]

$$\begin{aligned}E_{\mu\nu}\equiv\, & G_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\gamma_0(\Box)R-\frac{1}{2}g_{\mu\nu}R_{\alpha\beta}\gamma_2(\Box)R^{\alpha\beta}\\ &+2\frac{\delta R}{\delta g^{\mu\nu}}\gamma_0(\Box)R+\frac{\delta R_{\alpha\beta}}{\delta g^{\mu\nu}}\gamma_2(\Box)R^{\alpha\beta}+\frac{\delta R^{\alpha\beta}}{\delta g^{\mu\nu}}\gamma_2(\Box)R_{\alpha\beta}\\ &+\frac{\delta\Box^r}{\delta g^{\mu\nu}}\left[\frac{\gamma_0(\Box^l)-\gamma_0(\Box^r)}{\Box^r-\Box^l}R^2\right]\\ &+\frac{\delta\Box^r}{\delta g^{\mu\nu}}\left[\frac{\gamma_2(\Box^l)-\gamma_2(\Box^r)}{\Box^r-\Box^l}R_{\alpha\beta}R^{\alpha\beta}\right]=0,\end{aligned} \tag{4}$$

where $\Box^{l,r}$ act on the left and right arguments (on the right of the incremental ratio) as indicated inside the brackets.

From (4), one could find that the Ricci-flat solution to $R_{\mu\nu}=0$ is also an exact solution to $E_{\mu\nu}=0$ [7]. It is given by the Schwarzschild solution with line element

$$ds^2=\bar{g}_{\mu\nu}dx^\mu dx^\nu=-f(r)dt^2+\frac{1}{f(r)}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{5}$$

where $f(r)$ is the metric function defined by

$$f(r)=1-\frac{r_0}{r} \tag{6}$$

with the horizon radius (size) r_0. Furthermore, the Kerr metric, being another Ricci-flat solution to $R_{\mu\nu}=0$, is also an exact solution to the non-local gravity.

4. Perturbations: linearized equations

Now, let us derive the linearized equation from (4) for the case of $\gamma_2\neq 0$ by considering the perturbation $h_{\mu\nu}$ around the background metric tensor $\bar{g}_{\mu\nu}$,

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}, \tag{7}$$

where overbar ($\bar{}$) denotes the background spacetime. First of all, we would like to mention that the black hole solution obtained from (1) with $\gamma_2=0$ by choosing either (2) or (3) is stable against linear perturbations [8]. When choosing the case of (2), its linearized equation is reduced to

$$\delta R_{\mu\nu}(h)=0 \tag{8}$$

implying the stability of the Schwarzschild black hole in Einstein gravity [15–18]. Eq. (8) is indeed a second-order differential equation, which is solvable for $h_{\mu\nu}$.

On the other hand, for the case of (3), its linearized equations are composed of the two forms

$$\left(\bar{\Box}-M^2\right)\delta R(h)=0, \tag{9}$$

$$\delta R_{\mu\nu}(h)-\frac{1}{6}\bar{g}_{\mu\nu}\delta R(h)-\frac{1}{3M^2}\bar{\nabla}_\mu\bar{\nabla}_\nu\delta R(h)=0, \tag{10}$$

which are surely independent of the exponential form of entire function $e^{H(\bar{\Box})}$. Here, $\bar{\Box}$ is the background d'Alembertian defined by

$$\begin{aligned}\bar{\Box}=\bar{g}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu=&-\frac{1}{f(r)}\frac{\partial^2}{\partial t^2}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2f(r)\frac{\partial}{\partial r}\right)\\ &+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}.\end{aligned} \tag{11}$$

Thus, the linearized equations (9) and (10) are exactly the same one obtained from the local Starobinsky theory $\mathcal{L}_f=\sqrt{|g|}[R+R^2/(6M^2)]$ in Ref. [9]. Eq. (10) corresponds to a second-order equation for $\delta R(h)$ coupled to $\delta R_{\mu\nu}(h)$, which is not easy to be solved. In other words, Eq. (10) is a fourth-order equation for the metric perturbation $h_{\mu\nu}$. Therefore, the stability can be proved by introducing an auxiliary field at the level of the action (by lowering $\mathcal{L}_f$ to a second order scalar-tensor theory) before performing perturbation process [19].

For case of $\gamma_2\neq 0$, one may attempt to derive a more simpler equation of motion because Eq. (4) is too lengthy to analyze the stability of the black hole. Ignoring quadratic order in the Ricci tensor (**Ric**), one finds [7]

$$G_{\mu\nu}+2\frac{\delta R_{\alpha\beta}(g)}{\delta g^{\mu\nu}}\left[g^{\alpha\beta}\gamma_0(\Box)R+\gamma_2(\Box)R^{\alpha\beta}\right]+O(\mathbf{Ric}^2)=0. \tag{12}$$

We note that all the complicated incremental ratios in Eq. (4) are dropped out of Eq. (12) since these ratios are quadratic in the Ricci tensor. Considering the linear perturbation of $\delta R_{\mu\nu}(h)$, the replacement of $\bar{R}_{\mu\nu}=0$ cancels them out.

Imposing the unitarity condition of $\gamma_2(\Box)=-2\gamma_0(\Box)$ around the Minkowski spacetime, one reduces Eq. (12) to

$$G_{\mu\nu}+2\frac{\delta R_{\alpha\beta}(g)}{\delta g^{\mu\nu}}\gamma_2(\Box)G^{\alpha\beta}+O(\mathbf{Ric}^2)=0. \tag{13}$$

Using the relation

$$\frac{\delta R_{\alpha\beta}(g)}{\delta g^{\mu\nu}}=\frac{1}{2}g_{\alpha(\mu}g_{\nu)\beta}\Box+\frac{1}{2}g_{\mu\nu}\nabla_\alpha\nabla_\beta-g_{\alpha(\mu|}\nabla_\beta\nabla_{|\nu)}, \tag{14}$$

we obtain an equation of motion as

$$\begin{aligned}&G_{\mu\nu}+\Box\Big(\gamma_2(\Box)G_{\mu\nu}\Big)+\nabla_\alpha\nabla_\beta\Big(\gamma_2(\Box)G^{\alpha\beta}\Big)g_{\mu\nu}\\ &\quad-2g_{\alpha(\mu|}\nabla_\beta\nabla_{|\nu)}\Big(\gamma_2(\Box)G^{\alpha\beta}\Big)+O(\mathbf{Ric}^2)=0.\end{aligned} \tag{15}$$