Physics Letters B 779 (2018) 420–424

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Dirac field in the background of a planar defect

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ARTICLE INFO

Article history:
Received 4 February 2018
Accepted 15 February 2018
Available online 21 February 2018
Editor: M. Cvetič

ABSTRACT

We study massless Dirac fermions in the background of a specific planar topologically nontrivial configuration in the three-dimensional spacetime. The results show the presence of massive bound states, phase shifts and the consequent differential cross section for the scattering of fermions in the weak coupling regime. Despite the nontrivial topology of the background field, no fermionic zero mode is found.

1. Introduction

In general, the interaction of fermion fields with solitonic backgrounds, topological or nontopological, may create or affect various interesting physical phenomena like charge and fermion number fractionalization, vacuum polarization and Casimir effect, superconductivity, Bose–Einstein condensation, conducting polymers and localization of fermions in the braneworld scenarios (see for example [1–13]). Besides that, there are interesting works related to the investigation of fermions in soliton backgrounds in the context of supersymmetry (see for example [14–17]). The massless Dirac fermions emerge as the quasiparticles in various novel materials such as graphene and topological insulators exhibiting intriguing behaviors [18,19]. In the context of 2D materials like graphene, it is important to study the band structure and properties of the trapped Dirac electron states and the consequent electronic properties of the material in the presence of a defect (see for instance [11,20,21]).

In $2+1$ dimensions there are two particularly interesting types of solitons; vortices appearing in Maxwell–Higgs and Chern–Simons theories where one can attribute electric charge to the vortex in the latter case. For specific choices of the Higgs or scalar potential, the minimum energy static vortex solutions satisfy a set of first-order differential self-duality equations, or known as Bogomol'nyi equations. In [22] the authors introduced a new set of topological defects respecting self-duality condition in $2+1$ dimensions where the translational symmetry of the system is broken. In the models they considered, there is only one real scalar field which in general makes it impossible to have topological defects thanks to Derrick–Hobart no go theorem. The key point in their work to circumvent the obstruction was to introduce explicit space dependence in the potential term of the boson field. We think it would be interesting to study fermions in detail, considering the effect of symmetries/symmetry breakings, in a specific 2D nontrivial configuration with this characteristic.

The fermionic zero modes are relevant to the quantum theory of the models, while the zero modes of the bosonic fluctuations determine the collective coordinates that describe the solitons. However, in supersymmetric models, the fermionic zero modes can be directly related to the zero modes of bosonic fluctuations describing massless modes around the vortices. Fermionic zero modes in the Dirac equation for fermions coupled to a topologically nontrivial defect background are important in systems belonging to a large domain in physics, going from high energy to condensed matter physics. Specially their relation with the topology of the background defect is of considerable interest. In [23], Jackiw and Rossi showed that the Dirac field has $|n|$ zero modes in the n-vortex background field. This means that the fermionic zero modes are protected due to the nontrivial topology of the background soliton. In this line of work one can find large number of papers in the literature (see for example [24] and references therein). However, we discuss here a counter example when the system does not respect translational symmetry and the Lagrangian has explicit space dependence. Besides that the model does not contain a gauge field and the corresponding topological flux associated to the vorticity of the system. We show that in the model considered in this paper, there is no fermionic zero mode, although the background configuration produces a planar

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https://doi.org/10.1016/j.physletb.2018.02.035

topological structure that can be used to simulate a skyrmion-like structure with unity skyrmion number, a subject explored before in Refs. [25,26].

We studied fermions in the background of several 1D kinklike configurations in [27,28] and found the fermionic zero mode as well as all other massive bound spectrum. In the current paper, we consider a massless Dirac field in a specific rotationally symmetric and localized topological structure in $2+1$ dimensions where the system does not respect translational symmetry. We are interested in the fermionic bound energy spectrum as well as the scattering phase shift due to the interaction with the defect. When the coupling constant of the fermion–soliton interaction is small compared to the self-interaction of the boson field, resulting in a heavy soliton compared to the scales appearing in the system, it is possible to ignore the effect of the fermion on the soliton which is the case in this paper. In this sense we say the defect is the background perturbation for the Dirac field. Due to the presence of a solitonic background as trapping potential, the fermion field spectrum can be distorted, i.e., bound states can appear and continuum states can change as compared with the free fermion.

The work deals mainly with the fermionic bound energy spectrum besides the scattering phase shift in the presence of the background defect studied before in [22,25,26]. In this model we can see that there is no fermionic zero mode, although the background configuration is topologically nontrivial. In Sec. 2 we introduce the theory and discuss about the symmetries of the system, which we use to drive the simplified versions of the equations of motion. In Sec. 3 we briefly review a model that can be used to describe a skyrmion-like structure with unity skyrmion number. In the model to be considered here it is easy to see that there is no need to add a gauge field to have a well-defined theory, in contrast with the Maxwell–Higgs vortex model. Finally, in Sec. 4 we summarize and discuss the main results of the current work.

2. Yukawa coupling

The Lagrangian density adopted in the present work has the following form

$$\mathcal{L}=\bar{\psi}\, i\gamma^{\mu}\partial_{\mu}\psi-g\phi\,\bar{\psi}\psi+\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi-U(r;\phi), \tag{2.1}$$

where ψ and ϕ are fermion and boson fields, respectively. We work in $2+1$ dimensions and write the Lagrangian density in the form

$$\mathcal{L}=\mathcal{L}_b+\mathcal{L}_f, \tag{2.2}$$

where the bosonic contribution is given by [22]

$$\mathcal{L}_b=\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi-\frac{1}{r^2}V(\phi), \tag{2.3}$$

and

$$\mathcal{L}_f=\bar{\psi}\, i\gamma^{\mu}\partial_{\mu}\psi-g\phi\,\bar{\psi}\psi. \tag{2.4}$$

In this paper, we are interested in studying the fermion system given by the Lagrangian density $\mathcal{L}_f$ interacting with the background planar defect configuration, the solution of the equation of motion considering $\mathcal{L}_b$ with the scalar potential

$$V(\phi)=\frac{a}{2}(v^2-\phi^2)^2. \tag{2.5}$$

The fermion field couples to the bosonic structure via the Yukawa coupling parameter g, and the equation of motion, considering the Lagrangian $\mathcal{L}_f$, has the form

$$\left(i\gamma^{\mu}\partial_{\mu}-g\phi\right)\psi=0\,. \tag{2.6}$$

As it is clear from the equation of motion (2.6) the system breaks parity symmetry which is not unusual in $2+1$ dimensions, due to the fact that the parity symmetry acts differently and parity transformation should be taken as reflection in just one of the spatial axes. Besides that, the system does not have energy-reflection symmetry. Therefore, we do not expect symmetric energy spectrum around the line $E=0$. The representation we choose for the Dirac matrices is $\gamma^0=\sigma_3$, $\gamma^1=i\sigma_2$ and $\gamma^2=-i\sigma_1$. The charge-conjugation transformation is representation dependent. The system is symmetric under this transformation and in this specific representation the charge-conjugation operator is γ^2.

In $2+1$ dimensions, the mass dimension of the bosonic field ϕ, the spinor field ψ and the coupling constant g are $1/2$, 1 and $1/2$, respectively. We rescale all mass scales by the value of the field ϕ at infinity as $\phi\to\phi/v$, $\psi\to\psi/v^2$, $r\to rv^2$, $g\to g/v$ and $a\to av^2$. Therefore, from now on all parameters of the system are dimensionless.

We then define

$$\psi\equiv e^{-iEt}\begin{pmatrix}\psi_1(r,\theta)\\ \psi_2(r,\theta)\end{pmatrix} \tag{2.7}$$

in order to get the Dirac equation in the explicit form

$$\begin{aligned}\left(ie^{-i\theta}\partial_r+\frac{e^{-i\theta}}{r}\partial_\theta\right)\psi_2(r,\theta)&=-\left[E-g\phi(r)\right]\psi_1(r,\theta),\\ \left(ie^{i\theta}\partial_r-\frac{e^{i\theta}}{r}\partial_\theta\right)\psi_1(r,\theta)&=-\left[E+g\phi(r)\right]\psi_2(r,\theta).\end{aligned} \tag{2.8}$$

The rotation symmetry allows us to write down an ansatz for the solution to the Dirac equation using separation of variables

$$\begin{aligned}\psi_1(r,\theta)&=\psi_1(r)e^{i(j-1/2)\theta},\\ \psi_2(r,\theta)&=\psi_2(r)e^{i(j+1/2)\theta},\end{aligned} \tag{2.9}$$

where $\psi_1(r)$ and $\psi_2(r)$ are complex in general. Substituting the above relations in the equations of motion leads to

$$\begin{aligned}i\left(\partial_r+\frac{(j+1/2)}{r}\right)\psi_2(r)&=-\left[E-g\phi(r)\right]\psi_1(r),\\ i\left(\partial_r-\frac{(j-1/2)}{r}\right)\psi_1(r)&=-\left[E+g\phi(r)\right[\psi_2(r).\end{aligned} \tag{2.10}$$

This set of equations are not symmetric under $j\to-j$ which is reflecting the fact that the system does not respect parity. Separating imaginary and real parts of the components of the spinor field as

$$\begin{aligned}\psi_1(r)&=\psi_1^R(r)+i\psi_1^I(r),\\ \psi_2(r)&=\psi_2^R(r)+i\psi_2^I(r),\end{aligned} \tag{2.11}$$

results in

$$\begin{aligned}\left(\partial_r+\frac{(j+1/2)}{r}\right)\psi_2^I(r)&=\left(E-g\phi(r)\right)\psi_1^R(r),\\ \left(\partial_r-\frac{(j-1/2)}{r}\right)\psi_1^R(r)&=-\left(E+g\phi(r)\right)\psi_2^I(r),\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}\left(\partial_r+\frac{(j+1/2)}{r}\right)\psi_2^R(r)&=-\left(E-g\phi(r)\right)\psi_1^I(r),\\ \left(\partial_r-\frac{(j-1/2)}{r}\right)\psi_1^I(r)&=\left(E+g\phi(r)\right)\psi_2^R(r).\end{aligned} \tag{2.13}$$