Physics Letters B 779 (2018) 460–463

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Decay modes of the Hoyle state in ^{12}C

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ARTICLE INFO

Article history:
Received 12 November 2017
Received in revised form 1 February 2018
Accepted 16 February 2018
Available online xxxx
Editor: J.-P. Blaizot

Keywords:
Hoyle state
Efimov state
Bose Einstein condensate
Tunneling
Direct decay
Sequential decay

ABSTRACT

Recent experimental results give an upper limit less than 0.043% (95% C.L.) to the direct decay of the Hoyle state into 3α respect to the sequential decay into $^{8}\text{Be} + \alpha$. We performed one and two-dimensional tunneling calculations to estimate such a ratio and found it to be more than one order of magnitude smaller than experiment depending on the range of the nuclear force. This is within high statistics experimental capabilities. Our results can also be tested by measuring the decay modes of high excitation energy states of ^{12}C where the ratio of direct to sequential decay might reach 10% at $E^*(^{12}\text{C}) = 10.3$ MeV. The link between a Bose Einstein Condensate (BEC) and the direct decay of the Hoyle state is also addressed. We discuss a hypothetical 'Efimov state' at $E^*(^{12}\text{C}) = 7.458$ MeV, which would mainly *sequentially* decay with 3α of *equal energies*: a counterintuitive result of tunneling. Such a state, if it would exist, is at least 8 orders of magnitude less probable than the Hoyle's, thus below the sensitivity of recent and past experiments.

The changing shape of a nucleus depending on its excitation energy plays a very important role in nuclear reactions, such as the formation of ^{12}C, ^{16}O etc., essential for understanding the interiors of stars [1–4]. The linear chain structures constitute a longstanding challenge both in theoretical and experimental studies since Morinaga suggested this exotic arrangement in 1950s [5,6]. Antisymmetrized molecular dynamics (AMD) calculations, which allow the coexistence of cluster nuclei and shell-model-like aspects, predict a linear-like chain of α clusters, in which the largest angle of the vertices is more than 120 degree, for the 0_3^+, 1_1^-, 2_2^+ and 2_1^- at 10.3, 10.844, 11.16 and 11.828 MeV respectively. In these states, one α cluster seems almost to be escaping. While the 3α clusters in the 0_2^+, 3_1^- and 4_1^- states at 7.654, 9.641 and 19.55 MeV form isosceles triangle configurations close to equilateral triangles [7]. However, there have been no experimental confirmations for 3α chain in ^{12}C for those states. Recently, by studying the reaction $^{40}\text{Ca} + {}^{12}\text{C}$ at 25 MeV/nucleon, it has been suggested that the Hoyle state has a minor decay branch $7.5 \pm 4.0\%$ that produces three particles of almost equal energy and that such a decay provides evidence for Bose Einstein Condensate (BEC) [8]. Later, by studying different projectile-target and beam energy combinations, a much lower limit was set for the component with three nearly equal-energy α particles [9–12]. The two most recent experiments at the time of writing [11–13] give an upper limit for the ratio of the direct decay (DD) to sequential decay (SD) less than 0.043% (=1/2500, i.e., 1 out of 2500 events). The main reason for the difficulty to determine different decay modes of the Hoyle state is due to the experimental acceptance. In fact modern silicon or other detectors type have a resolution in energy of the order of tens of keV. This number should be compared to the width of the Hoyle state, which is 8.5 eV [14]. For instance, Raduta et al. [8] claim a $17.0 \pm 5.0\%$ decay rate into the DD, but the *measured* width of their state is 330 keV due to the detector sensitivity. Increasing the detector sensitivity and the experimental technique results in a width of about 50 keV and corresponding decrease of the ratio to less than 0.5% [15] and further improved the upper limit to 0.2% [16], improving on the first result by Freer et al. [17], less than 4%. The latest improvements in Refs. [11–13], where higher statistics data was collected and further cuts were adopted by selecting events having an excitation energy narrowly distributed around the Hoyle state centroid at 7.65 MeV, gave values of 0.047% [12] and 0.043% [11] respectively. Clearly, the more cuts/constraints are adopted the larger the statistics must be in order to obtain a more precise value for the DD. The lower limit quoted above corresponds to about 2500 events in the region of interest.

It is important to stress that the configurations found in microscopic calculations might not be recovered experimentally since the excited nuclei, in order to decay, have to tunnel the Coulomb

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https://doi.org/10.1016/j.physletb.2018.02.040

barrier. Because of the tunneling, for instance for the Hoyle state, the condition where two α particles are close together to form a ^{8}Be while the other α tunnels the barrier is more favorable. This is essentially the configuration (SD) found in experiments [8,10–13,15,17]. We will now discuss the conditions for which a configuration (say SD) is more favorable respect to another (say DD). Let us first recall some known features of ^{8}Be decay. This nucleus is short lived and decays into two alphas. The process can be understood through quantum mechanical tunneling. The two α-particles acquire about 91 keV after the decay. We can approximate the decay using the Gamow prescription, i.e., neglecting the nuclear attraction. This approximation is rather good when the final kinetic energies of the fragments are very small as compared to the Coulomb barrier. However, corrections due to the nuclear force can be easily implemented assuming a step function [18]. The probability of tunneling at finite impact parameter, is given in terms of the action A [19,20]:

$$\Pi[E - \frac{l(l+1)\hbar^2}{2I}] \sim e^{\frac{2}{\hbar}A};\ A = \int_{R_N}^{R_0} dr\sqrt{2\mu(V_c(r) - E)}, \tag{1}$$

where $V_c(r)$, R_N and R_0 are the Coulomb potential, the inner and the outer classical turning points, respectively. E, l, I and μ are the center of mass energy, the angular momentum, the moment of inertia and the reduced mass. In particular, if we consider the pure Coulomb penetrability, i.e., we take the limit of $R_N \to 0$, $l = 0$, and no nuclear force, A becomes $A_G = Z_T Z_P e^2 \pi\sqrt{\frac{\mu}{2E}}$: the Gamow limit. The lifetime is given by:

$$\tau = \tau_B(1 + e^{\frac{2}{\hbar}A}). \tag{2}$$

The $\Gamma_{1/2}$ width is given by $\Gamma_{1/2} = \frac{\hbar}{\tau \ln 2}$. The characteristic time for assault to the barrier τ_B is left as a free parameter and fitted to the lifetime of ^{8}Be. Its value could be estimated from the average velocity of nucleons in the nucleus and its radius or other means, but since we are mainly concerned with ratios, τ_B cancels out. Also, to avoid a detailed description of the nuclear force, we assume that it overcomes the Coulomb repulsion for distances below $R_N = \alpha_N \times (R_1 + R_2)$ and we vary the value of α_N from zero (Gamow limit) to 1 (touching spheres of radii $R_{1,2}$). As we will show, the ratio is smoothly dependent on the value of α_N and future experimental results might confirm or disprove it. In order to reduce the variation of the ratio further, we can approximate the Coulomb potential to that of overlapping spheres when the relative distance between the separating nuclei is less than $R(= R_1 + R_2)$

$$V_c(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R}(3 - \frac{r^2}{R^2}), & \text{if } r \le R; \\ \frac{Z_1 Z_2 e^2}{R}, & \text{if } r \ge R. \end{cases} \tag{3}$$

This term gives a modest modification of the results (at most a factor 2.5 for the Hoyle state in the Gamow limit) thus giving more confidence to our approach and it will not be considered further. Eqs. (1) and (2) can be easily applied to the decay of ^{8}Be and the SD of the Hoyle state. In the more general case of the Hoyle decay directly into 3α, the action must be generalized in order to take into account the fact that tunneling is now occurring in the plane determined by the 3 particles, i.e., in two dimensions. We follow the spirit of Refs. [19,20] by defining collective variables **P** and **R**. In particular, in the case of equal energies of the 3 tunneling αs the problem simplifies further since the relative momenta (distances) of the 3 particles are equal. The action becomes $A = \int_{R_N}^{R_0} d\mathbf{RP}$ which in two dimensions gives a factor of 2 after integrating over the relative angle between **P** and **R**, thus effectively reducing the two dimensional problem to a one dimensional as in Eq. (1). Thus the action is calculated with the condition $r_{12} = r_{13} = r_{23} = r$ and similarly for the momenta. It is clear that different decay modes, for instance a linear chain, can be easily calculated with the conditions $r_{12} = r_{13} = r_{23}/2$, i.e., with particle 1 at the center of the chain. But these decay modes are also negligible respect to the SD. We expect a change less than a factor of 2 adding more configurations. On the other hand, the ratios of DD/DDE for the Hoyle state are around 4 from different experimental measurements [9,16]. DDE represents the three emitted αs with similar energies in DD. Therefore, in this paper we will discuss the equal energy decays only. As we will see later, the ratio of DD/SD in our calculation is 40 times less than the experimental limit, our conclusion will not be affected by this simplification in our framework.

Under these assumptions it is straightforward to solve the action integral for different values of α_N and different excitation energies of ^{12}C and angular momenta. In particular we have solved Eqs. (1) and (2) for the Hoyle, the 9.641 MeV ($J^\pi = 3^-$) and the 10.3 MeV ($J^\pi = 0^+$) states. As mentioned in the introduction, to these states we have added a hypothetical 'Efimov state (ES)' [21,22] at $E^* = 7.458$ MeV, $J^\pi = 0^+$. One of the reasons for discussing this state is on the question of the BEC explored by Raduta et al. [8]. The authors claim that equal energies of the 3α might be a signature for a BEC. We argue that such a claim could be not even true for non-interacting bosons since below the critical temperature we might have Bosons with zero energy and Bosons with energy larger than zero. We will show that for the ES the decay is mostly sequential, as for the Hoyle state, but the three αs have the same final energy as for the DD! The fact that the 3α have the same energy gives a precise value of the excitation energy of the ES. Recall that the ES arises when no two body bound state exists but just a strong resonance while the three body system gets strongly bound [23–26]. This result is based on the Thomas theorem discussed in the 30′ [22] and might apply well to α since ^{8}Be is not bound while ^{12}C is. Exploiting this fact one can define an ES when three particles are in mutual resonance with each other [24,25,27], the vice versa might not be true [27]. This mechanism is similar to the exchange of a pion, which produces the nuclear force. The difference is that one of the 3 component particles is exchanged between the two others. In other words an α is exchanged between the other two and the relative kinetic energy of the 3α is exactly 91.84 keV($\times 3$), i.e., the gs of ^{8}Be for each couple. The exchange of the α particle results in a $1/R^2$ attractive potential only effective for $l = 0$. This phenomenon has been demonstrated in atomic physics [26,28] but not so far in nuclear physics for which it was first predicted. The main difficulty (apart Coulomb) is the short-range nuclear interaction and the scattering length, which are comparable. If we assume that the 3α are in mutual resonance to form ^{8}Be, we can obtain the excitation energy of the state as:

$$\begin{aligned} E^*(ES) &= \frac{2}{3}\sum_{i=1,j>i}^{i=3} E_{ij} - Q = \frac{2}{3}(0.09184 \times 3) + 7.2747 \text{ MeV} \\ &= 7.458 \text{ MeV}. \end{aligned} \tag{4}$$

We would like to stress that we are not advocating here the existence of the ES but just show how tunneling could produce counterintuitive results. Detailed three-body calculations [31,35] show a sharp resonance for the Hoyle state and a kink or bump depending on the used model in the region of the suggested ES.