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Genuine and effective actions, the Master Equation and Suppressed SUSY

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ABSTRACT

Genuine theories are defined to be those governed by a Master Equation. All other theories are defined to be Effective Theories. It is straightforward to integrate heavy particles out of a Genuine Theory to get an Effective Theory, but putting together a Genuine Theory takes years. The Minimal Supersymmetric Standard Model is an Effective Theory, which arises because of the hypothesis of an invisible sector where spontaneous breaking of Supersymmetry (SUSY) occurs. Suppressed SUSY allows us to construct a Genuine Theory for SUSY, without the need for spontaneous breaking of SUSY.

1. **Some history:** In 1969 a big textbook emerged, about the Weak Interactions [1]. It discussed the current–current Fermi type interaction and other issues at length. Probably the main puzzle in quantum field theory, around that time, was whether there could be a sensible theory of massive vector bosons. Many thousands of papers later, the Standard Model of Weak, Electromagnetic and Strong interactions had evolved [2]. Grand Unified Theories certainly seemed to unify some things, but there were still plenty of problems [3,4].

2. **Definition of Genuine Theory:** The Standard Model and the Grand Unified Theories all had Master Equation formulations [5–13]. Let us call all theories that have Master Equation formulations 'Genuine Theories', and all other theories 'Effective Theories'. Thus, the current–current Fermi theory is an 'Effective Theory' and the Standard Model and the Grand Unified Theories are Genuine Theories.

3. **A one way street:** While it is fairly simple to go from a Genuine Theory to a related Effective Theory [14], it is not at all easy to go in the other direction, as was noted above in paragraph 1. One can integrate 'auxiliary fields' and keep the Master Equation quite easily. But integrating fields that require the inversion of non-trivial differential operators loses the Master Equation.

4. **Complicated Genuine Theories with SUSY and Gravity:** While it was obvious that Gravity and Supergravity theories would also have a Master Equation form, this did not seem to add any useful insight to the Standard Model or Grand Unified Theories.

5. **Genuine Theories and Effective Theories:** Theories with gravity benefited from the Master Equation in that the higher derivative quadratic terms with $\Box^n$ could be absorbed into the lower terms [15]. But when one integrates the graviton (or any heavy particle) out of the theory, as in [16] say, the Master Equation necessarily disappears. Thus for example the original current–current theory can be obtained from the Standard Model by integrating out the vector bosons and the Higgs boson. But when that is done, then the Master Equation is lost, because it needs these bosons to be in the theory to formulate the Master Equation in the usual way.

6. **Return to Effective Theories:** Theories with SUSY obviously had the problem that SUSY needed to be broken, and this led rapidly to the issue of the 'invisible sector' and the 'messenger sector'. Once these were viewed as essential, nothing useful could be extracted from the Master Equation, because the Action was, by assumption, no longer known, or even testable. The Minimal Supersymmetric Standard Model ('MSSM') grew from this [17,18], and it was clearly a kind of Effective Theory. That theory has a 'mass cutoff' at a mass characterizing spontaneous breaking of SUSY.

7. **Loose Language, Genuine Theories and Effective Theories:** At some point it became popular to say that all theories were Effective Theories. Now one difference between Effective Theories and Genuine Theories is that Effective Theories have a 'mass cutoff' which can be thought of as resulting from the 'integrating out' of some massive particles in the theory. On the other hand, Genuine Theories do not have mass cutoffs. They are valid to an arbitrary number of loops and incorporate the Master Equation to keep the symmetry to all orders. They are renormalizable in the sense that even if there are an infinite number of independent mass scales, the theory maintains the symmetry that it starts with. There is no

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such constraint for Effective Theories. So it is necessary to be careful to note that adding effective terms spoils the Master Equation.

8. **Another possibility:** The idea behind Suppressed SUSY is that we can make a new Genuine Theory by using an Exchange Transformation on a simpler Genuine Theory. Now clearly this will only be interesting if we can find an alternative way to 'break SUSY' that does not involve spontaneous supersymmetry breaking, because that leads to the hidden sector, etc. which leads to the Minimal Supersymmetric Standard Model, which necessarily has a cutoff, and loss of the symmetry.

9. **Suppressed SUSY:** So the question is whether there might be some way to keep the Master Equation for a SUSY theory, while still getting rid of the problem that supermultiplets are degenerate in mass. This is in fact possible! Preliminary discussions appear in [19,20], but, as pointed out there, the more correct discussion requires Supergravity to be part of the theory. Here we construct an Exchange Transformation in the SU(5) Grand Unified Theory. In a sequel paper [21], we use this example with Supergravity.

10. **The Higgs multiplets must be doubled in SUSY theories:** The feature that makes the SUSY case special is the fact that, as is well known [22–29], one needs to have a 2 and also a $\overline{2}$ SU(2) doublet of Higgs to have a chance of generating masses for both the up and down quarks in all versions of the Supersymmetric Standard Model, whereas the Standard Model requires only one SU(2) doublet of Higgs. Similarly, one needs to have both a 5 and a $\overline{5}$ Higgs representation in the SU(5) Grand Unified Theory coupled to Supergravity, for a similar reason. The reason is simple–SUSY requires that the Yukawa interactions, and the quark masses, in these theories, emerge from three chiral superfields multiplied together, with a VEV for the Higgs. In the non-SUSY Standard Model or non-SUSY Grand Unified Theory cases one can simply use one Higgs and its complex conjugate to get masses for both the up and down quark masses. But this violates SUSY in SUSY theories, since if the Higgs is chiral then its Complex Conjugate is not. So the up and down quark masses must come from the VeVs of two different SUSY Higgs-type multiplets in a SUSY theory.

11. **Taking an SSM type theory part way back to the SM using an Exchange Transformation to generate Suppressed SUSY:** In [19,20] we looked at this issue with a 2 and also a $\overline{2}$ SU(2) doublet of Higgs for a kind of SSM, without worrying very much about Supergravity. As will now be seen, we can start with a 5 and a $\overline{5}$, of SU(5), and perform an Exchange Transformation which effectively makes the theory look more like the Standard Model, which has only one of these 5 Higgs (and its complex conjugate). This peculiar situation gets very interesting when put into an SU(5) Grand Unified Supergravity Theory. It results in a SU(5) Grand Unified Supergravity Theory with Suppressed SUSY, which is a sort of mixture of an SU(5) Grand Unified Theory and an SU(5) Grand Unified Supergravity Theory, with some nice properties from both of them. There are many ways to do this, depending on the choice of the Exchange Transformation. We focus on one version here for simplicity.

12. **Contents of this paper:** This paper starts with a detailed short review and derivation of the Master Equation for a special Yang–Mills Theory with a special set of scalar representations (the 'Old Theory'). Then in Paragraph 25, we implement the creation of a New Theory, from the Old Theory, by using a special 'Exchange Transformation', which looks like a canonical transformation for the Master Equation, except that it interchanges some New and Old Fields with some New and Old Sources. The Master Equation is a kind of Grassmann odd Poisson Bracket. For a Poisson Bracket in classical mechanics, coordinates and momenta are equivalent under a canonical transformation [30,31]. However Fields and Sources are NOT EQUIVALENT for the Master Equation in the Quantum Field Theory, because Fields are quantized, and Sources are not. The Quantum Field Theory arises from path integration of the Fields, leaving the Sources unintegrated. If the Fields change, then the Quantum Field Theory changes too. In Paragraph 27, we set out the New Action. We derive the New Theory from the New Action, and then the New Master Equation in Paragraph 31. A Summary of details, with the Old and New Field Actions, is in Paragraph 32, and the issues arising from implementing the same Exchange Transformation in Supergravity are discussed in the Conclusion in Paragraph 33.

13. **The old action with 68 real scalars:** We take a simple model with $SU(5)$ Yang–Mills coupled to scalars, without any Supersymmetry whatsoever. For the scalars we choose three special representations H_L^i, H_{Ri}, S^a because these are also important for Suppressed SUSY in [21]. H_L^i and H_{Ri} are in the fundamental 5 and $\overline{5}$ representations. S^a is in a $24+24$ (complex adjoint) representation.

14. **Hermitian matrices to generate SU(5):** As is well known, the simplest way to deal with the group SU(5), in the Yang–Mills context [3,4], is to use the notation

$$T_j^{ai};\, i=1\cdots 5;\, j=1\cdots 5;\, a=1\cdots 24, \tag{1}$$

for the 24 (5×5) hermitian generators of $SU(5)$ in the fundamental 5×5 representation. Here are some results for Complex Conjugation $*$, Transposition T and Adjoint $\dagger$:

$$\left(T_j^{ai}\right)^* = \left(T_j^{ai}\right)^T = T_i^{aj};$$
$$\left(T_j^{ai}\right)^\dagger \equiv \left(\left(T_j^{ai}\right)^*\right)^T \equiv \left(\left(T_j^{ai}\right)^T\right)^* = T_j^{ai} \tag{2}$$

The matrices T_j^{ai} are also traceless. The commutation relations are:

$$T_j^{ai}\delta_i^j = T_i^{ai} = 0;\, \delta_i^i = 5;\, \left[T^a, T^b\right] = i f^{abc} T^c;\; T^a = (T^a)^\dagger \tag{3}$$

where we can choose to define $(T^aT^b)_i^j = T_k^{aj}T_i^{bk}$. Here we use the Kronecker δ_j^i for the unit matrix and f^{abc} is totally antisymmetric.

15. **Notation and indices:** The multiplet H_L^i transforms in the same way as the multiplet $\overline{H}_R^i$. Complex conjugation has the effect:

$$\left(H_L^i\right)^* = \overline{H}_{Li};\; (H_{Ri})^* = \overline{H}_R^i \tag{4}$$

Here we also have a complex adjoint S^a representation. This S^a is really two representations in the present context, since we could choose it to be real in this Yang–Mills SU(5) context. We can also choose the matrices to satisfy the following:

$$S_j{}^i = S^a T_j^{ai};\; T_j^{ai}T_i^{bj} = 2\delta^{ab};\; S_i{}^j T_j^{ai}; = 2S^a;\; \mathrm{Tr}[S^2] = 2S^aS^a; \tag{5}$$

$$(S^a)^* = \overline{S}^a;\; \left(S_j^i\right)^* = \overline{S}_i^j = \overline{S}^a T_i^{aj};\; \overline{S}_i{}^j T_j^{ai}; = 2\overline{S}^a;\; \mathrm{Tr}[S\overline{S}] = 2S^a\overline{S}^a \tag{6}$$

16. **Gauge covariant derivatives:** We define the gauge covariant derivative by:

$$D_{\mu j}{}^i = \left(\partial_\mu \delta_j^i - iV_\mu^a T_j^{ai}\right) \tag{7}$$

In the above V_μ^a is the SU(5) Gauge Yang–Mills Field, and we absorb the coupling constant g_5, which is easy to restore. We raise the Lorentz index by:

$$D_j^{\mu i} = \left(\partial^\mu \delta_j^i - iV^{a\mu} T_j^{ai}\right) \tag{8}$$