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Pure natural inflation

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ABSTRACT

We point out that a simple inflationary model in which the axionic inflaton couples to a pure Yang–Mills theory may give the scalar spectral index (n_s) and tensor-to-scalar ratio (r) in complete agreement with the current observational data.

Cosmic inflation plays an important role in explaining observable features of our universe, including its extreme flatness, as well as the origin of primordial curvature perturbations. The detailed predictions of inflation, however, depend on the potential $V(\phi)$ of the inflaton field ϕ. An important issue, therefore, is to understand what is the correct model of inflation and how it emerges from the underlying physics.

Recent observations by Planck [1] and BICEP2/Keck Array [2] have started constraining simple models of inflation. In particular, arguably the simplest model of inflation $V(\phi) = m^2\phi^2/2$ [3]—which gives the correct value for the scalar spectral index $n_s \simeq 0.96$—is now excluded at about the 3σ level because of the non-observation of tensor modes. This raises the following questions. Does the model of inflation need to be significantly complicated? Is the agreement of n_s of the quadratic potential with the data purely accidental?

In this letter, we argue that the answers to these questions may both be no. In particular, we argue that a simple inflationary model in which the inflaton ϕ couples to the gauge field of a pure Yang–Mills theory

$$\mathcal{L} = \frac{1}{32\pi^2}\frac{\phi}{f}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,F_{\mu\nu}F_{\rho\sigma}, \tag{1}$$

may give the values of n_s and the tensor-to-scalar ratio, r, in perfect agreement with the current observational data. Here, ϕ is a pseudo-Nambu–Goldstone boson—axion—of a shift symmetry $\phi \to \phi + \text{const.}$, and f is the axion decay constant. For now we assume that the gauge group of the Yang–Mills theory is $SU(N)$, but the model also works for other gauge groups; see later.

Conventionally, the potential of the axion field as in Eq. (1) is assumed to take the form generated by non-perturbative instantons

$$V(\phi) = \Lambda^4\left[1 - \cos\left(\frac{\phi}{f}\right)\right], \tag{2}$$

where Λ is the dynamical scale of the Yang–Mills theory. The resulting inflation model is called natural inflation [4,5], which has been extensively studied in the literature. The potential of Eq. (2), however, is not favored by the current data, and it would soon be excluded at a higher confidence level if the bound on r improves with n_s staying at the current value; see Fig. 1.

It is known since long ago, however, that the cosine potential in Eq. (2) is not correct in general, as argued by Witten [6,7] in the large N limit [8] with the 't Hooft coupling $\lambda \equiv g^2 N$ held fixed.[1] In particular, while the physics is periodic in ϕ with the period of $2\pi f$ (because $\theta \equiv \phi/f$ is the θ angle of the Yang–Mills theory), the multi-valued nature of the potential allows for the potential of ϕ in a single branch

$$V(\phi) = N^2\Lambda^4\,\mathcal{V}\left(\frac{\lambda\phi}{8\pi^2 N f}\right), \tag{3}$$

not to respect the periodicity under $\phi \to \phi + 2\pi f$. Here, the combination

$$x \equiv \frac{\lambda\phi}{8\pi^2 N f}, \tag{4}$$

appearing in the argument of $\mathcal{V}(x)$ is determined by analyzing the large N limit. This allows for building axionic models of inflation

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[1] See, e.g., Refs. [9–13] for related discussion in the context of inflation.

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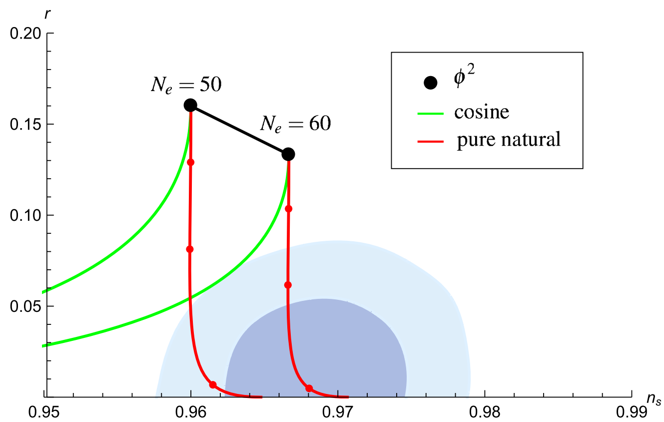

Fig. 1. The predicted values of n_s and r superimposed with the 68% and 95% CL BICEP2/KECK Array contours in Ref. [2]. The black dots represent the predictions of the quadratic potential $V(\phi) = m^2\phi^2/2$, with e-folding $N_e = 50$ and 60. The green lines are the predictions of the cosine potential, Eq. (2), and the red lines are those of the (holographic) pure natural inflation potential of Eq. (11). For the latter, we have varied $F/M_{\rm Pl} = 0.1$–100, with $F/M_{\rm Pl} = 10, 5, 1$ indicated by the red dots (from top to bottom). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in which the range of the field excursion exceeds the decay constant f [14–16].

The potential of Eq. (3) has an expansion of the form

$$V(\phi) = \sum_{n=1}^{\infty} b_{2n} \left(\frac{\phi}{F}\right)^{2n}, \tag{5}$$

where $F \propto f$. The values of the coefficients b_{2n}—more precisely their signs and double ratios—are important for how the predictions for n_s and r change as F is varied. If the cosine potential in Eq. (2) were valid, then we would obtain

$$\mathrm{sgn}(b_{2n}) = (-1)^{n-1}, \tag{6}$$

and

$$\frac{\frac{b_6}{b_4}}{\frac{b_4}{b_2}} = \frac{2}{5}, \qquad \frac{\frac{b_8}{b_6}}{\frac{b_6}{b_4}} = \frac{15}{28}, \qquad \cdots, \tag{7}$$

which lead to the curves labeled as "cosine" in Fig. 1. The correct values of the double ratios, however, are expected to be different from these values. In fact, b_{2n}'s obtained by lattice gauge theory disfavor the cosine form of Eq. (2) and are rather consistent with those expected from large N expansion [17].

While b_{2n}'s may in principle be determined by lattice calculations, their errors are still large. Instead, we may infer the form of the potential by the following arguments. First, invariance under the CP transformation $\phi \to -\phi$ implies that $\mathcal{V}(x)$ is a function of x^2, where we have absorbed a possible bare θ parameter in the definition of ϕ. Second, $\mathcal{V}(x)$ is expected to flatten as the potential energy approaches the point of the deconfining phase transition with increasing $|\phi|$ (since the dynamics generating the potential will become weaker). Assuming that the potential is given by a simple power law, we thus expect $\mathcal{V}(x) \sim 1/(x^2)^p$ $(p > 0)$. This potential is singular at $x \to 0$, and a simple way to regulate it is to replace x^2 with $x^2 + \text{const}$. After setting the minimum of the potential to be zero, these considerations give

$$V(x) = M^4 \left[1 - \frac{1}{(1 + cx^2)^p}\right] \quad (p > 0), \tag{8}$$

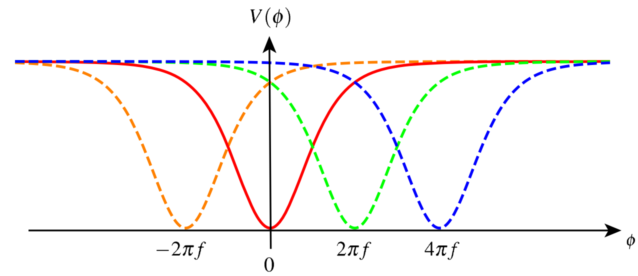

Fig. 2. The potential of pure natural inflation (in the holographic limit $p = 3$); Eq. (11). The potentials for other branches, which ensure the periodicity of physics under $\phi \to \phi + 2\pi f$, are also depicted by dashed lines.

where $M \sim \sqrt{N}\Lambda$, and $c > 0$ is a parameter of order unity. Here, we have used the well-established fact that the coefficient of x^2 is positive when $\mathcal{V}(x)$ is expanded around $x = 0$. We call the model of inflation in which the axionic inflaton potential is generated by a pure Yang–Mills theory (whose potential we expect to take the form of Eq. (8)) *pure natural inflation.*

As in the cosine potential, the potential of Eq. (8) gives $\mathrm{sgn}(b_{2n}) = (-1)^{n-1}$. It, however, gives different values of the double ratios

$$\frac{\frac{b_6}{b_4}}{\frac{b_4}{b_2}} = \frac{2(p+2)}{3(p+1)}, \quad \cdots, \quad \frac{\frac{b_{2n+4}}{b_{2n+2}}}{\frac{b_{2n+2}}{b_{2n}}} = \frac{(n+1)(p+n+1)}{(n+2)(p+n)}, \quad \cdots. \tag{9}$$

Therefore, predictions of this model are different from those of conventional natural inflation. (For example, by equating $(b_6/b_4)/(b_4/b_2)$ we obtain $p = -7/2 < 0$.) Here, we have assumed that the effect of a transition between different branches can be neglected, which we will argue to be the case.

The potential of Eq. (8) can be obtained by a holographic calculation [10,18], which is applicable in the limit of large N and 't Hooft coupling. In this calculation, N D4-branes in type IIA string theory are considered, with the D4-branes wrapping a circle. Below the Kaluza–Klein scale $M_{\rm KK}$ for the circle, the theory reduces to a 4d (non-supersymmetric) pure $SU(N)$ Yang–Mills theory, with the dynamical scale

$$\Lambda = M_{\rm KK}\, e^{-\frac{24\pi^2}{11\lambda}}, \tag{10}$$

where λ is the 't Hooft coupling at $M_{\rm KK}$. Considering the backreaction to the geometry of the constant Wilson line of the Ramond–Ramond one-form, which represents the θ angle of the gauge theory, the potential of the form of Eq. (8) is obtained with $c = 1$ and $p = 3$. Specifically, the potential of ϕ for a single branch is given by

$$V(\phi) = M^4 \left[1 - \frac{1}{\left(1 + \left(\frac{\phi}{F}\right)^2\right)^3}\right], \tag{11}$$

where[2]

$$M^4 = \frac{\lambda N^2}{3^7 \pi^2} M_{\rm KK}^4, \qquad F = \frac{8\pi^2 N}{\lambda} f. \tag{12}$$

The potentials for the other branches are obtained by replacing ϕ with $\phi + 2\pi k f$ $(k \in \mathbb{Z})$; see Fig. 2.

[2] The 't Hooft coupling (and the gauge coupling squared) defined in Ref. [10] is a factor of 2 smaller than λ (g^2) here.