Physics Letters B 775 (2017) 63–70

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On the ultimate uncertainty of the top quark pole mass

M. Beneke [a], P. Marquard [b], P. Nason [c], M. Steinhauser [d,*]

[a] Physik Department T31, James-Franck-Straße, Technische Universität München, D-85748 Garching, Germany
[b] Deutsches Elektronen Synchrotron DESY, Platanenallee 6, D-15738 Zeuthen, Germany
[c] INFN, Sezione di Milano Bicocca, 20126 Milan, Italy
[d] Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany

ARTICLE INFO

Article history:
Received 18 May 2016
Received in revised form 4 June 2017
Accepted 23 October 2017
Available online 26 October 2017
Editor: G.F. Giudice

ABSTRACT

We combine the known asymptotic behaviour of the QCD perturbation series expansion, which relates the pole mass of a heavy quark to the $\overline{\text{MS}}$ mass, with the exact series coefficients up to the four-loop order to determine the ultimate uncertainty of the top-quark pole mass due to the renormalon divergence. We perform extensive tests of our procedure by varying the number of colours and flavours, as well as the scale of the strong coupling and the $\overline{\text{MS}}$ mass. Including an estimate of the internal bottom and charm quark mass effect, we conclude that this uncertainty is around 110 MeV. We further estimate the additional contribution to the mass relation from the five-loop correction and beyond to be around 300 MeV.

1. Introduction

The top quark mass is a fundamental parameter of the Standard Model (SM). Due to its large size, it has non-negligible impact in the precision tests of the SM. After the discovery of the Higgs boson and the measurement of its mass, the values of the W and top mass are strongly correlated, such that a precise determination of both parameters would lead to a SM test of unprecedented precision [1]. Indeed, there is presently some tension between the value of the top mass 177 ± 2.1 GeV fitted from electroweak data and from its direct measurement [1], for which the combination of the Tevatron and LHC data yields the 1.6 σ lower value of $173.34 \pm 0.27 \pm 0.71$ GeV [2]. The value of the top mass is also crucial to the issue of stability of the SM vacuum (see [3] for a recent analysis). The Higgs quartic coupling decreases at high scales, eventually becoming negative. This evolution is very sensitive to the top mass value. For example, a top mass near 171 GeV would imply that the quartic coupling may vanish at the Planck scale, rather than turn negative.

The standard direct determination of the top mass at hadron colliders, being based upon observables that are related to the mass of the system comprising the top decay products, are quoted as measurements of the pole mass. On the other hand, it seems more natural to use the $\overline{\text{MS}}$ mass in both precision electroweak observables and in vacuum stability studies. In [4] the relation between the $\overline{\text{MS}}$ and pole mass for a heavy quark (the "mass conversion formula" from now on) has been computed to the fourth order in the strong coupling α_s. Assuming the value of 163.643 GeV for the top-quark $\overline{\text{MS}}$ mass $\overline{m}_t = m_t(\overline{m}_t)$, and assuming $\alpha_s^{(6)}(m_t) = 0.1088$, we have [4]

$$m_P = 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005)\ \text{GeV} \tag{1.1}$$

for the series expansion of the mass conversion formula. The last term from the fourth order correction is less than one half of the third order one.

It is also known that the mass conversion formula is affected by infrared (IR) renormalons [5–7]. This means that there are factorially growing terms of infrared origin in the perturbative expansion, such that the expansion starts to diverge at some order. If the series is treated as an asymptotic expansion, the ambiguity in its resummation is of order of a typical hadronic scale. Because of this, it is often stated that the ultimate accuracy of top pole mass cannot be below a few hundred MeV. One of the goals of this work is to make this estimate more precise.

It is remarkable that the perturbative relation between the pole and $\overline{\text{MS}}$ mass of a heavy quark appears to be dominated by the leading infrared renormalon already in low orders [8,9]. This observation was used in previous work [10–12], and more recently

* Corresponding author.
E-mail address: matthias.steinhauser@kit.edu (M. Steinhauser).

https://doi.org/10.1016/j.physletb.2017.10.054

in [14,15] to estimate the unknown normalization of the leading IR renormalon, and mostly applied in the context of bottom physics. In the context of top physics, the importance of this issue was raised recently in [16]. The purpose of this work is to combine the newly available four-loop coefficient [4] in the mass conversion formula with the known structure of the first infrared renormalon singularity [7] to determine the normalization constant and discuss its impact on top physics. We also perform an analysis of the dependence on the number of colours and flavours, which is by itself of interest, and stability tests with respect to variations of the scale of the strong coupling and $\overline{\text{MS}}$ mass. This leads to an expression for the mass conversion factor including an estimate of the contributions beyond four loops, and an estimate of the irreducible error.

2. Reminder

The renormalon divergence is a manifestation of the fact that the mass conversion formula, while infrared finite is sensitive to small loop momentum. In the case of the pole mass this sensitivity is particularly strong, namely linear, resulting in rapid divergence of the perturbative expansion, and an infrared sensitivity of order Λ_{QCD} [5,6]. The ambiguity in defining the pole mass is therefore of similar size. This is not surprising as the pole mass of a quark is not an observable due to confinement and the difference with the physical heavy meson masses is also of order Λ_{QCD}. Unlike other heavy quarks, the top quark decays on hadronic time scales, and thus the propagator pole position acquires an imaginary part. The renormalon divergence is not altered [17] by the fact that the top quark is unstable with a width larger than Λ_{QCD} and hence does not form bound states. The finite width simplifies the perturbative treatment of top quarks, since it provides a natural IR cut-off, and there exists no quantity for which the pole mass would ever be relevant. But the infrared sensitivity of the QCD corrections to the mass conversion factor, which causes the divergence, remains unaffected by the width.

Slightly more technically, the divergence arises from logarithmic enhancements of the loop integrand. Heuristically, this can be understood by noticing that the running coupling evaluated at the scale l of the loop momentum has the expansion

$$\alpha_s(l) = \frac{1}{b_0 \ln l^2/\Lambda_{\text{QCD}}^2} = \frac{\alpha_s(m)}{1-\alpha_s(m) b_0 \ln m^2/l^2} = \sum_1^{\infty} \alpha_s^n(m)\, b_0^n \ln^n \frac{m^2}{l^2}\,. \tag{2.1}$$

The IR contribution to the last loop integration in the $(n+1)$-loop order then takes the form

$$\delta m^{(n+1)} \propto \alpha_s^{n+1}(m) \int^m \mathrm{d}l\, b_0^n \ln^n \frac{m^2}{l^2} = m\,(2b_0)^n\, \alpha_s^{n+1}(m)\, n!\,. \tag{2.2}$$

With this behaviour the series of mass corrections reaches a minimal term of order

$$\begin{aligned} m\,(2b_0)^n \alpha_s^{n+1} n! &\approx m\, \alpha_s\, n^{-n} \left(\sqrt{2\pi} n^{n+1/2} e^{-n}\right) \\ &\approx m \sqrt{\frac{\pi \alpha_s}{b_0}} \exp\left(-\frac{1}{2b_0\alpha_s}\right) \\ &\approx \sqrt{\frac{\pi\alpha_s}{b_0}}\, \Lambda_{\text{QCD}}, \end{aligned} \tag{2.3}$$

when $n \approx 1/(2b_0\alpha_s)$ and then diverges. Asymptotic expansions can sometimes be summed using the Borel transform. Given a power series

$$f(\alpha_s) = \sum_{n=1}^{\infty} c_n \alpha_s^n\,, \tag{2.4}$$

the corresponding Borel transform is defined by

$$B[f](t) = \sum_{n=0}^{\infty} c_{n+1} \frac{t^n}{n!}\,. \tag{2.5}$$

The Borel integral

$$\int_0^{\infty} dt\, e^{-t/\alpha_s}\, B[f](t) \tag{2.6}$$

has the same series expansion as $f(\alpha_s)$ and provides the exact result under suitable conditions. However, for the case of (2.2), where $c_{n+1} = (2b_0)^n n!$, the Borel integral

$$\int_0^{\infty} dt\, e^{-t/\alpha_s} \frac{1}{1-2b_0 t} \tag{2.7}$$

cannot be performed because of the pole at $t = 1/(2b_0)$. We can introduce some prescription for handling the pole in the integral, as, for example, the principal value prescription. Whether or not this reconstructs the exact result, an ambiguity remains, quantified by the imaginary part of the integral when going above or below the singular point. A commonly used procedure is to define this ambiguity to be equal to the imaginary part of the integral divided by Pi (see, e.g., [18], section 5.2). For (2.7), this yields

$$\Lambda_{\text{QCD}}/(2b_0)\,. \tag{2.8}$$

In the range of α_s values considered in this paper, the ambiguity is close to the size of the smallest term in (2.3).[1]

It can be shown [7] that while the precise asymptotic behaviour of the mass conversion formula differs from the simple ansatz employed in this section for illustration, as discussed below, the ambiguity is exactly proportional to Λ_{QCD}, which evaluates to about 250 MeV in the $\overline{\text{MS}}$ scheme. In the remainder of this work, we aim to quantify the proportionality factor.

3. The leading pole mass renormalon

We write the perturbative expansion of the mass conversion formula as

$$m_P = m(\mu_m)\left(1 + \sum_{n=1}^{\infty} c_n(\mu, \mu_m, m(\mu_m))\, \alpha_s^n(\mu)\right). \tag{3.1}$$

Here $\alpha_s(\mu)$ is the $\overline{\text{MS}}$ coupling in the n_l light flavours theory, and $m(\mu_m)$ stands for the $\overline{\text{MS}}$ mass evaluated at the scale μ_m. (In the following we will consider different scale choices for the heavy quark mass and the strong coupling. We also use $\overline{m}$ to denote the $\overline{\text{MS}}$ mass evaluated self-consistently at a scale equal to the mass itself, i.e.

$$\overline{m} = m(\overline{m}). \tag{3.2}$$

[1] Note, however, the different parametric dependence on α_s of (2.3) and (2.8). The correct dependence is that of (2.8), for the following reason: The typical width of the region where the minimal term is attained grows parametrically as $\sqrt{1/(2b_0\alpha_s)}$. The accuracy of an asymptotic series is better estimated by the minimal term times the factor accounting for the number of terms in this region, which makes (2.3) parametrically consistent with (2.8). Numerically, this factor turns out to be of order one for the applications considered in this paper, as will be confirmed in section 4 below. In case of doubt, the estimate from the ambiguity of the Borel integral should be the preferred choice.