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Repulsive baryonic interactions and lattice QCD observables at imaginary chemical potential

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ABSTRACT

The first principle lattice QCD methods allow to calculate the thermodynamic observables at finite temperature and imaginary chemical potential. These can be compared to the predictions of various phenomenological models. We argue that Fourier coefficients with respect to imaginary baryochemical potential are sensitive to modeling of baryonic interactions. As a first application of this sensitivity, we consider the hadron resonance gas (HRG) model with repulsive baryonic interactions, which are modeled by means of the excluded volume correction. The Fourier coefficients of the imaginary part of the net-baryon density at imaginary baryochemical potential – corresponding to the fugacity or virial expansion at real chemical potential – are calculated within this model, and compared with the $N_t = 12$ lattice data. The lattice QCD behavior of the first four Fourier coefficients up to $T \simeq 185$ MeV is described fairly well by an interacting HRG with a single baryon–baryon eigenvolume interaction parameter $b \simeq 1$ fm^3, while the available lattice data on the difference $\chi_2^B - \chi_4^B$ of baryon number susceptibilities is reproduced up to $T \simeq 175$ MeV.

1. Introduction

The Monte Carlo lattice QCD simulations provide the equation of state of the $(2+1)$-flavor strongly interacting matter at zero chemical potential [1–3]. A crossover-type transition is observed [4]. The pseudocritical temperature T_{pc} of the transition depends on the observable used to define it, estimates based on chiral condensate and its susceptibility give $T_{pc} \approx 155$ MeV [5,6], while observables based on strangeness suggest somewhat higher temperatures [5,7]. Below the transition one expects to find the confined hadronic phase. Many lattice QCD observables in that temperature range are indeed well described by a simple ideal hadron resonance gas (HRG) model [7–10].

It was pointed out recently, that the behavior of lattice observables in the crossover region, particularly of correlations and fluctuations of conserved charges, is very sensitive to the modeling of the baryonic interactions [11,12]. This sensitivity is of great interest, since hadronic modeling of conserved charge fluctuations is often used to extract freeze-out parameters of heavy ion collisions [13,14]. Lattice observables at finite net baryon density can certainly be expected to be even more sensitive to the modeling of these interactions. Unfortunately, direct Monte Carlo calculations at finite μ_B are hindered by the sign problem. Main methods to circumvent this problem include the reweighting techniques [15–18], the Taylor expansion around $\mu = 0$ [19–22], and the analytic continuation from imaginary μ [23–31]. These methods have allowed to calculate some thermodynamic features of QCD at small but finite chemical potentials [32–34].

In the present work we consider the imaginary μ method. We use the updated version of the lattice data, shown previously in Ref. [35]. However, instead of performing analytic continuation from imaginary chemical potential to real chemical potential, we instead directly compare lattice data at imaginary μ to the

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corresponding predictions of the phenomenological models. Some phenomenological models were considered at imaginary chemical potential before, such as the quasiparticle model [36] or the PQM model [37]. Comparison of phenomenological models with lattice simulations at imaginary μ was considered in [24,38–40]. In the present work, our focus is on the HRG model with repulsive interactions for baryon–baryon and antibaryon–antibaryon pairs, modeled by means of the excluded volume (EV) correction.

The paper is organized as follows: in Sec. 2 the lattice observables at imaginary baryochemical potential, which are studied in the present work, are introduced. Sec. 3 lists the predictions for these observables from several phenomenological models. The lattice method is described in Sec. 4, and in Sec. 5 lattice results are compared to the predictions of interacting HRG models. Summary in Sec. 6 closes the article.

2. QCD observables at imaginary baryochemical potential

Due to the baryon–antibaryon symmetry, the QCD pressure is an even function of a real baryochemical potential μ_B at a finite temperature. This quantity can then be written as the following series expansion:

$$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cosh(k\,\mu_B/T), \tag{1}$$

provided that the expansion is convergent at a given T-μ_B pair.[1] At $\mu_B = 0$, the pressure is simply the sum of all coefficients $p_k(T)$. Therefore these can be interpreted as the partial pressures, coming from the sectors of the Hilbert space with a different baryon number.

The first-order net baryon susceptibility $\chi_1^B(T,\mu_B) \equiv \partial(p/T^4)/\partial(\mu_B/T)$ is proportional to the net baryon density and it is equal to

$$\chi_1^B(T,\mu_B) = \frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T) \sinh(k\,\mu_B/T), \tag{2}$$

where, by definition,

$$b_k(T) \equiv k\, p_k(T). \tag{3}$$

It is clear that the knowledge of all $p_k(T)$ coefficients provides complete information about the thermodynamic properties of QCD in the region of the phase diagram where the series expansion given by Eq. (1) is convergent.

One can consider the susceptibility χ_1^B in Eq. (2) at a purely imaginary value of the baryochemical potential, i.e. at $\mu_B = i\,\tilde{\mu}_B$. The analytic continuation yields

$$\chi_1^B(T, i\tilde{\mu}_B) = i \sum_{k=1}^{\infty} b_k(T) \sin(k\,\tilde{\mu}_B/T), \tag{4}$$

i.e. χ_1^B itself becomes purely imaginary. The imaginary part of χ_1^B in Eq. (4) has explicit form of the trigonometric series expansion, with $b_k(T)$ being the corresponding temperature dependent Fourier coefficients. If the $\tilde{\mu}_B$-dependence of χ_1^B is known (e.g. from lattice simulations), then the coefficients $b_k(T)$ can be calculated in the standard way:

$$b_k(T) = \frac{2}{\pi} \int_0^{\pi} d\tilde{\mu}_B\, [\mathrm{Im}\, \chi_1^B(T, i\tilde{\mu}_B)] \sin(k\,\tilde{\mu}_B/T). \tag{5}$$

[1] Throughout this work we assume that strangeness and electric charge chemical potentials are zero, i.e. $\mu_S = \mu_Q = 0$.

3. Phenomenological models

In some analytic models of the equation of state, the coefficients $b_k(T)$ can be worked out explicitly.

3.1. Ideal HRG

A popular model to describe the confined phase of QCD at low temperatures is the hadron resonance gas model. In its simplest implementation, the system is modeled as a non-interacting mixture of all known hadrons and resonances. It is argued [41], that the inclusion into the model of all known resonances as free non-interacting (point-like) particles, may allow for an effective modeling of the attractive interactions between hadrons, including the formation of narrow resonances and of Hagedorn states. This ideal HRG model has a long history of being used to describe the hadron production in heavy-ion collisions at various collision energies [42–46].

In the present HRG analysis we employ the Boltzmann approximation for all baryons. This is a good approximation for the observables of interest. We do not include the light nuclei into the HRG particle list. The inclusion of nuclei would induce nonzero $b_2, b_3, \ldots$, but always with a positive sign. This is in contrast to our lattice results, e.g. that $b_2 < 0$, indicating that the next important correction to the HRG model is not from these states, but from repulsive interactions. The net baryon density ρ_B^{id} in the ideal HRG model reads

$$\rho_B^{\mathrm{id}}(T,\mu_B) = 2\,\phi_B(T) \sinh(\mu_B/T), \tag{6}$$

where

$$\phi_B(T) = \sum_{i \in B} \int dm\, \rho_i(m)\, \frac{d_i\, m^2\, T}{2\pi^2}\, K_2\left(\frac{m}{T}\right) \tag{7}$$

is the baryonic spectrum, with d_i and ρ_i being, respectively, the degeneracy and a properly normalized mass distribution for hadron type i, and where the sum goes over all baryons in the system. Note that the summation does not include antibaryons. We include the baryon states, which are listed in the Particle Data Tables [47] and have a confirmed status there. The function ρ_i takes into account the non-zero widths of the resonances by the additional integration over their Breit–Wigner shapes, following Refs. [48,49].

It is evident from Eq. (6) that all Fourier coefficients b_k^{id} are equal to zero for $k \geq 2$. For the first coefficient one obtains $b_1^{\mathrm{id}}(T) = 2\,\phi_B(T)/T^3$.

3.2. HRG with repulsive baryonic interactions

In a more realistic HRG model one has to also take into account the attractive and repulsive interactions between hadrons which cannot be attributed to the resonance formation. In particular, the nucleon–nucleon interaction is known to be largely repulsive at short distances and the corresponding scattering phase shifts are not known to exhibit any resonance structure. The importance of the van der Waals like interactions between baryons for lattice QCD observables was recently pointed out in Ref. [11]. In the present work we perform similar analysis for the observables at imaginary chemical potential. To keep things simple, we focus on the short-range repulsion between baryons.

Following Refs. [11,50] we assume that repulsive interactions exist between all baryon–baryon and antibaryon–antibaryon pairs. These interactions are modeled by means of the excluded-volume (EV) correction [51]. At the same time, the EV interactions between all other hadron pairs are explicitly omitted. It is not clearly established whether significant EV-type interactions exist between