Physics Letters B 776 (2018) 270–277

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Electrodynamics of dual superconducting chiral medium

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ARTICLE INFO

Article history:
Received 4 September 2017
Received in revised form 26 November 2017
Accepted 26 November 2017
Available online 1 December 2017
Editor: J.-P. Blaizot

ABSTRACT

We study the electrodynamics of the chiral medium with electric and magnetic charges using the effective Maxwell–Chern–Simons theory extended to include the magnetic current. The exchange of helicity between the chiral medium and the magnetic field, known as the inverse cascade, is controlled by the chiral anomaly equation. In the presence of the magnetic current, the magnetic helicity is dissipated, so that the inverse cascade stops when the magnetic helicity vanishes while the chiral conductivity reaches a non-vanishing stationary value satisfying $\sigma_\chi^2 < 4\sigma_e\sigma_m$, where σ_e, σ_m and σ_χ are the electric, magnetic and chiral conductivities respectively. We argue that this state is superconducting and exhibits the Meissner effect for both electric and magnetic fields. Moreover, this state is stable with respect to small magnetic helicity fluctuations; the magnetic helicity becomes unstable only when the inequality mentioned above is violated.

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1. Introduction

Classical electromagnetic field in a medium with chiral anomaly is described by a system of Maxwell equations and the chiral anomaly equation [1,2] known as the Maxwell–Chern–Simons (MCS) theory [3–6]. The chiral anomaly equation controls the exchange of helicity between the field and medium such that the total helicity is conserved. The resulting non-trivial evolution of the magnetic field topology has been a subject of recent interest [7–18] motivated by phenomenological applications in nuclear physics, condensed matter physics and cosmology [19].

A distinctive feature of the MCS theory is the emergence of the soft magnetic field modes exponentially growing in time [7,8,14,15,19–28]. These unstable modes transfer helicity from the medium to the field in a process known as the inverse cascade [8,29]. Eventually, however, the helicity conservation puts a cap on the inverse cascade [30,31].

It has been argued in [32–36] that magnetic monopoles play an important role in quark–gluon plasma dynamics. Magnetic monopoles also often appear in cosmological models [37] and even in condensed matter physics [38]. This motivates us to consider the MCS theory with dynamical magnetic monopoles (MCSm). That the magnetic monopoles are expected to have non-trivial effects on the magnetic field can be seen from the fact that the dual transformation generates in the Lagrangian the same CP-odd term as chiral anomaly. In particular, the magnetic current, while being energy non-dissipative, causes dissipation of the total helicity. The main goal of this paper is to uncover the main features of the chiral magnetic dynamics with magnetic monopoles.

The paper is organized as follows. In Sec. 2 we formulate the equations of the MCSm theory and analyze their main properties. Our main assumption is the linear medium response that is characterized by the electric and magnetic conductivities σ_e and σ_m. We observe the emergence of the superconducting phase when $\sigma_\chi^2 < 4\sigma_e\sigma_m$ and formulate the corresponding London equations (12), (13) in Sec. 2.2. In Sec. 2.3 we analyze the late-time dynamics of the MCSm system, in particular, its evolution towards a stationary state. We argue that the magnetic helicity must exponentially decay due to the helicity dissipating magnetic current. The chiral conductivity σ_χ also decays owing to the inverse cascade as mentioned above. However, in the presence of the magnetic current, the inverse cascade may be terminated before the chiral conductivity turns zero. Therefore, the chiral conductivity approaches a finite stationary value σ_∞ while the magnetic helicity is completely dissipated. In Sec. 3 we investigate the dispersion relation of the magnetic field modes and point out the conditions under which the magnetic field (and magnetic helicity) is unstable. In our context, the term "instability" means that a small fluctuation of the field triggers its exponential growth, even though eventually it decays as a result of the magnetic helicity non-conservation. We

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https://doi.org/10.1016/j.physletb.2017.11.063
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show that the stability condition coincides with the condition for the existence of the superconductivity. In order to develop a clearer understanding of the time evolution of the magnetic field and the chiral conductivity, we employ in Sec. 4 the Fastest Growing State (FGS) model [30] which assumes that the magnetic helicity at later times is driven by a mode with the exponentially largest growth rate. Using this model we perform in Sec. 5 a detailed investigation of the time-evolution of the MCSm theory. We argue that after undergoing an inverse cascade the system settles to the superconducting phase. This is the main result of our paper. We conclude with a discussion in Sec. 6.

2. Maxwell–Chern–Simons theory with magnetic monopoles

2.1. Maxwell and the chiral anomaly equations

A plasma of electric and magnetic charges with chiral anomaly is governed by the following generalization of the Maxwell equations [3–6]:

$$\nabla \cdot \boldsymbol{B} = 0\,, \tag{1}$$

$$\nabla \cdot \boldsymbol{E} = 0\,, \tag{2}$$

$$-\nabla \times \boldsymbol{E} = \partial_t \boldsymbol{B} + \boldsymbol{j}_m\,, \tag{3}$$

$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j}_e + \sigma_\chi \boldsymbol{B}\,, \tag{4}$$

where $\boldsymbol{j}_m$ is the magnetic current density and σ_χ is assumed to depend only on time. We neglected the electric and magnetic polarization of the plasma, which is a small effect for good conductors and consider the plasma to be electrically and magnetically neutral. Assuming the linear response $\boldsymbol{j}_e = \sigma_e \boldsymbol{E}$, $\boldsymbol{j}_m = \sigma_m \boldsymbol{B}$ with constant electric and magnetic conductivities we can derive, using (1)–(4), an equation for the magnetic field[1]

$$-\nabla^2 \boldsymbol{B} + \partial_t^2 \boldsymbol{B} = -(\sigma_e + \sigma_m)\partial_t \boldsymbol{B} - \sigma_e\sigma_m \boldsymbol{B} + \sigma_\chi(t)\nabla \times \boldsymbol{B}\,. \tag{5}$$

In view of (1) we can introduce the vector potential $\boldsymbol{A}$ as $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. Since the Bianchi identity is violated in the presence of the magnetic current, the relationship between the electric field and the vector potential is modified as compared to the Maxwell theory. One can check that

$$\boldsymbol{E} = -\partial_t \boldsymbol{A} - \sigma_m \boldsymbol{A}\,, \tag{6}$$

satisfies the modified Faraday's law (3) in the Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$. We note that the vector potential $\boldsymbol{A}$ obeys the same equation (5) as the magnetic field.

The relationship (6) between the electric field and the vector potential is not unique. One can add on its right-hand-side a gradient of any scalar function ϕ. The choice of ϕ is dictated by the requirement of the gauge-invariance of (6). Equations such as (6) appear in the theory of the superconductivity and indicate the necessity to introduce the magnetic monopole condensate. The condensate contributes to the right-hand-side of (6) a term proportional to the gradient of its phase ϕ which restores the gauge invariance. The term $-\sigma_m \boldsymbol{A}$ in (6) and the term proportional to $\nabla\phi$ make up the supercurrent. Not surprisingly, the supercurrent induces the Meissner effect discussed in the next sub-section. Throughout the paper we assume the gauge condition $\phi = 0$ (the unitary gauge).

[1] Magnetic field is supposed to be not very strong, so that the Larmor radius is much larger than the Debye radius r_D, which guarantees that the kinetic coefficients do not depend on B. For relativistic plasmas at temperature T this amounts to $eB \ll r_D T$.

The time-evolution of the chiral conductivity is governed by the chiral anomaly equation. At high temperatures it can be written as [9,30]

$$\partial_t \sigma_\chi = c_A^2/(\chi V)\int \boldsymbol{E}\cdot\boldsymbol{B}\, d^3x\,, \tag{7}$$

where $c_A = N_c \sum_f q_f^2 e^2/(2\pi^2)$ is the anomaly coefficient, V is the volume of the system and χ is the susceptibility that does not depend on time [30,39]. Eq. (7) can be written in terms of the magnetic helicity defined as

$$\mathcal{H}_{\rm em} = \int \boldsymbol{A}\cdot\boldsymbol{B}\, d^3x\,. \tag{8}$$

Denoting $\beta = c_A^2/(V\chi)$ yields

$$\beta^{-1}\partial_t\sigma_\chi = -\partial_t \mathcal{H}_{\rm em} - 2\sigma_m \mathcal{H}_{\rm em}\,. \tag{9}$$

Evidently, the total helicity $\mathcal{H}_{\rm tot} = \beta^{-1}\sigma_\chi + \mathcal{H}_{\rm em}$ is no longer a conserved quantity at finite σ_m [30]. While the magnetic current is energy non-dissipative, it does dissipate the magnetic helicity.

2.2. Meissner effect

That the magnetic current does not dissipate energy can also be seen from the fact that under time-reversal $\mathcal{T}$ the current density and magnetic field change signs, implying that the magnetic conductivity σ_m is even under $\mathcal{T}$. The same argument indicates that the chiral conductivity σ_χ is also even under $\mathcal{T}$, which, as recently argued by Kharzeev, implies the existence of the "chiral magnetic superconductivity" [40].

To see how the supercurrent induces the Meissner effect, it is convenient to introduce the "normal" and "super" components of the electric field as

$$\boldsymbol{E}_n = -\partial_t \boldsymbol{A}\,, \qquad \boldsymbol{E}_s = -\sigma_m \boldsymbol{A}\,. \tag{10}$$

We denote the electric currents induced by each component as

$$\boldsymbol{j}_n = \sigma_e \boldsymbol{E}_n\,, \qquad \boldsymbol{j}_s = \sigma_e \boldsymbol{E}_s = -\sigma_e\sigma_m \boldsymbol{A}\,. \tag{11}$$

It can be checked that both currents satisfy the continuity equation: $\nabla\cdot\boldsymbol{j}_n = \nabla\cdot\boldsymbol{j}_s = 0$. It is straightforward to see that the super current $\boldsymbol{j}_s$ satisfies the London equations:

$$\nabla \times \boldsymbol{j}_s = -\sigma_e\sigma_m \boldsymbol{B}, \tag{12}$$

$$\partial_t \boldsymbol{j}_s = +\sigma_e\sigma_m \boldsymbol{E}_n, \tag{13}$$

which indicates that $\boldsymbol{j}_s$ is indeed a superconducting current. The MCSm equations (1)–(4) can be rewritten for the pair of fields $\boldsymbol{B}$, $\boldsymbol{E}_n$ as

$$\nabla \cdot \boldsymbol{E}_n = 0, \tag{14}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{15}$$

$$-\nabla \times \boldsymbol{E}_n = \partial_t \boldsymbol{B}, \tag{16}$$

$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E}_n + \frac{\sigma_e + \sigma_m}{\sigma_e}\boldsymbol{j}_n + \boldsymbol{j}_s + \sigma_\chi \boldsymbol{B}. \tag{17}$$

In the stationary limit $\boldsymbol{j}_n = 0$, $\boldsymbol{E}_n = 0$ (12) and (17) yield

$$\nabla^2 \boldsymbol{B} = \sigma_e\sigma_m \boldsymbol{B} - \sigma_\chi \nabla \times \boldsymbol{B}\,, \tag{18}$$

which can also be seen directly from (5). The super component of the electric field satisfies the same equation. Indeed, taking the Laplacian of the second equation in (10) and using (17) we obtain

$$\nabla^2 \boldsymbol{E}_s = \sigma_e\sigma_m \boldsymbol{E}_s - \sigma_\chi \nabla \times \boldsymbol{E}_s\,. \tag{19}$$

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