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Color-suppression of non-planar diagrams in bosonic bound states

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ABSTRACT

We study the suppression of non-planar diagrams in a scalar QCD model of a meson system in $3+1$ space–time dimensions due to the inclusion of the color degrees of freedom. As a prototype of the color-singlet meson, we consider a flavor-nonsinglet system consisting of a scalar-quark and a scalar-antiquark with equal masses exchanging a scalar-gluon of a different mass, which is investigated within the framework of the homogeneous Bethe–Salpeter equation. The equation is solved by using the Nakanishi representation for the manifestly covariant bound-state amplitude and its light-front projection. The resulting non-singular integral equation is solved numerically. The damping of the impact of the cross-ladder kernel on the binding energies are studied in detail. The color-suppression of the cross-ladder effects on the light-front wave function and the elastic electromagnetic form factor are also discussed. As our results show, the suppression appears significantly large for $N_c=3$, which supports the use of rainbow-ladder truncations in practical non-perturbative calculations within QCD.

1. Introduction

Relativistic studies of non-perturbative systems are essential in order to understand the properties of composite systems, which are characterized by strong coupling constant and/or small constituent masses [1]. One relevant example is the calculation of masses and form factors of light mesons, consisting of quarks with small masses and gluons. The light-front two-body bound-state equation [1] was extended to the full light-front dynamic kernel including the ladder, cross-ladder, stretched-box, and particle–antiparticle creation/annihilation effects to study the contributions of higher Fock states [2] and its link to the powerful numerical approach known as the Feynman–Schwinger representation (FSR) approach [3,4] as well as the manifestly covariant Bethe–Salpeter formulation [5] has been discussed extensively.

An analytically tractable tool for the bound-state problem, what is also known as the most orthodox tool for dealing with the relativistic two-body problem in quantum field theory, is the Bethe–Salpeter equation (BSE), proposed in the 1950s [5], utilizing the Green's functions of covariant perturbation theory (see also [6]). This equation has for a long time been solved by a transformation to Euclidean space, by using an analytical continuation to the complex-energy plane [7]. Solutions in Minkowski space are however essential to perform calculations of structure-dependent observables such as form factors both in spacelike and timelike regions. One of the first solutions of the BSE was obtained by Kusaka and Williams in Ref. [8], who studied the bound-state system of two scalar bosons interacting through the exchange of another boson, by using the Nakanishi integral representation (NIR, [9]) and the ladder kernel in the lowest approximation. Another major improvement regarding the treatment of the Bethe–Salpeter equation in Minkowski space was performed by Karmanov and Carbonell in Ref. [10]. They studied the two-boson bound-state system in the ladder approximation, by using the NIR and the explicitly covariant Light-Front formalism [11]. They reformulated the BSE in a more convenient form and the ladder plus cross-ladder kernel was subsequently considered in Ref. [12].

In [13] Gigante et al. studied the impact of the cross-ladder contribution on the bound-state structure of the two-boson system in great detail. The elastic electromagnetic form factor was also computed by considering the two-current contribution in addition to the one given by the impulse approximation. It was found that the cross-ladder contribution to the kernel has a rather large impact on the coupling constants as well as the asymptotic behavior of the light-front wave functions, i.e. the coupling constant changed by about 30–50% depending on the considered binding energy.

Differently from this theory, in quantum chromodynamics (QCD) one has also to consider the color degrees of freedom. It was

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shown in [14] that the non-planar diagrams, like e.g. the cross-ladder, have a vanishing contribution in the limit $N_c \to \infty$, where N_c is the number of colors. $SU(N)_c$ theories of QCD in $1+1$ dimensions have been extensively studied by 't Hooft [15] and also by Hornbostel et al. [16].

In this work, we consider a scalar QCD model to study a prototype of a flavor nonsinglet meson system of a scalar-quark and a scalar-antiquark with equal masses exchanging a scalar-gluon of different mass in $3+1$ dimensions. This is obtained by adding the appropriate color factors in the manifestly covariant BSE and its light-front projection. We study quantitatively the suppression of the contribution coming from the cross-ladder interaction kernel in the nonperturbative problem of the color singlet two-boson bound state mass and structure, due to the color factors in the kernel for different number of colors $N_c = N$ with $N = 2, 3, 4$. The effects of the non-planar cross-ladder interaction kernel on the coupling constant for a given bound state mass as well as in light-front wave function, and the elastic electromagnetic form factor are studied in detail.

This work is organized as follows. In Sec. 2 we briefly introduce the Bethe–Salpeter equation and the Nakanishi integral representation. The formalism for introducing the color factors is also presented. Then, in Sec. 3 we discuss our numerical results for the coupling constants for a given bound state mass, the light-front wave function and the elastic electromagnetic form factor. Finally, in Sec. 4 we summarize our work and give an outlook.

2. Theoretical framework

2.1. Bethe–Salpeter equation

The Bethe–Salpeter equation (BSE) in Minkowski space, for two spinless particles, reads:

$$\Phi(k,p) = S(\eta_1 p + k) S(\eta_2 p - k) \int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p), \tag{1}$$

where the propagators $S(p')$ are in general dressed and can be represented by the Källen–Lehmann spectral representation as

$$S(p') = \int_0^\infty d\gamma \frac{\rho(\gamma)}{p'^2 - \gamma + i\epsilon}, \tag{2}$$

which raise up the following

$$\Phi(k,p) = \int_0^\infty d\gamma \frac{\rho(\gamma)}{(\eta_1 p + k)^2 - \gamma + i\epsilon} \int_0^\infty d\gamma' \frac{\rho(\gamma')}{(\eta_2 p - k)^2 - \gamma' + i\epsilon} \times \int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p), \tag{3}$$

where in the interaction kernel, $K(k,k',p)$, the exchanged boson is in general also dressed. For a prototype of flavor nonsinglet meson system, $K(k,k',p)$ does not get complicated by the annihilation process, but it keeps the usual ladder and cross-ladder irreducible kernels, which have been extensively discussed in Refs. [2] and [13]. The idea of our paper is to start exploring the effects of color degrees of freedom on the non-planar diagrams in the simplest possible way, without considering dressed propagators. For that purpose we put $\rho(\gamma) = \delta(\gamma - m^2)$, which gives

$$\Phi(k,p) = \frac{i^2}{\left[(\frac{p}{2}+k)^2 - m^2 + i\epsilon\right]\left[(\frac{p}{2}-k)^2 - m^2 + i\epsilon\right]} \times \int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p), \tag{4}$$

where the interaction kernel K is given by a sum of irreducible Feynman diagrams and considering the equal partition for the momentum fraction $\eta_1 = \eta_2 = 1/2$. The ladder kernel is considered in most of works, but here we also incorporate the cross-ladder contribution.

The Bethe–Salpeter (BS) amplitude can be found in the form of the Nakanishi integral representation [8,9]:

$$\Phi(k,p) = -i\int_{-1}^{1} dz \int_0^\infty d\gamma \frac{g(\gamma,z)}{\left[\gamma + m^2 - \frac{1}{4}M^2 - k^2 - p\cdot k\, z - i\epsilon\right]^3}. \tag{5}$$

The weight function $g(\gamma,z)$ is non-singular, whereas the singularities of the BS amplitude are fully reproduced by this integral. The BS amplitude in the form (5) is then substituted into the BS equation (4) and after some mathematical transformations, namely upon integration in $k^- = k^0 - k^3$ on both sides of the BS equation, one obtains the following non-singular integral equation for $g(\gamma,z)$ [13]:

$$\int_0^\infty \frac{g(\gamma',z)d\gamma'}{\left[\gamma' + \gamma + z^2m^2 + (1-z^2)\kappa^2\right]^2} = \int_0^\infty d\gamma' \int_{-1}^{1} dz'\, V(\gamma,z,\gamma',z')g(\gamma',z'), \tag{6}$$

where for bound states $\kappa^2 = m^2 - \frac{1}{4}M^2 > 0$ and V is expressed in terms of the kernel K [12].

Furthermore, the s-wave valence light-front wave function can be computed from

$$\Psi_{LF}(\gamma,z) = \frac{1-z^2}{4}\int_0^\infty \frac{g(\gamma',z)d\gamma'}{[\gamma' + \gamma + z^2m^2 + (1-z^2)\kappa^2]^2}, \tag{7}$$

derived in Ref. [10].

2.2. Scalar QCD

The extension of the BSE to a scalar QCD model is obtained simply by introducing the color matrices in the interaction vertices appearing in the kernel diagrams. The Feynman rules are first used and then the kernel dependence on N is derived by performing the trace of the product of Gell-Mann matrices, corresponding to a colorless composite two-boson system. For the ladder kernel this computation is straightforward:

$$tr[(\lambda^a)_{ji}(\lambda^a)_{ij}] = \sum_a (\lambda^a)_{ji}(\lambda^a)_{ij} = \frac{1}{2}\sum_{i,j=1}^{3}\left(\delta_{jj}\delta_{ii} - \frac{1}{N}\delta_{ji}\delta_{ij}\right) = \frac{1}{2}\left(N^2 - \frac{1}{N}\sum_{i=1}^{3}\delta_{ii}\right) = \frac{N^2-1}{2}, \tag{8}$$

where the internal boson line factors have been replaced by the corresponding $SU(N)$ projection operators (see Ref. [17]). As can be seen in Fig. 1, the color factors are given by $(\lambda_a)_{ij}(\lambda_a)_{kj'}$ and $(\lambda_b)_{jk}(\lambda_b)_{j'i}$, and they read