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Radiative corrections to the average bremsstrahlung energy loss of high-energy muons

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ABSTRACT

High-energy muons can travel large thicknesses of matter. For underground neutrino and cosmic ray detectors the energy loss of muons has to be known accurately for simulations. In this article the next-to-leading order correction to the average energy loss of muons through bremsstrahlung is calculated using a modified Weizsäcker–Williams method. An analytical parametrisation of the numerical results is given.

1. Introduction

The muon bremsstrahlung cross section has been studied extensively for many years [1–5]. Together with the production of electron–positron pairs [6–9] and the inelastic nuclear interaction [10–12] it describes the dominant contribution to the energy loss of high-energy muons.

Muons with energies of tens to hundreds of TeV can travel distances of the order of several kilometers. Therefore it is necessary to know the average energy loss per unit length

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = N \int E v \frac{\mathrm{d}\sigma}{\mathrm{d}v} \mathrm{d}v \tag{1}$$

accurately. Here $v = (E - E')/E$ is the relative energy loss per interaction, and N is the number density of target atoms. Previous calculations took into account the modification of the Coulomb interaction with the nucleus by elastic and inelastic nuclear form factors, the contribution of atomic electrons as target for muon bremsstrahlung and the inelastic interaction with the target nucleus. This article discusses the correction of the energy loss through virtual and real radiative corrections. Since this correction is small compared to the main contribution, we restrict our treatment of the nucleus to elastic atomic and nuclear form factors.

The energy loss is of importance for underground detectors for two reasons: on the one hand, the energy loss is needed to predict the spectrum of muons that will reach the detector; on the other hand the energy lost by a muon inside the detector on a given length is used to reconstruct the energy of the radiating particle. The energy reconstruction is further complicated by its sensitivity to the distribution of energy losses and their correlation to the energy of the muon. Especially rare large stochastic energy losses enlarge the variance of the energy loss per unit length. As a first step to revisit this problem, in the present article, the muon energy dependent average energy loss per length is calculated.

In the calculation of radiative corrections in QED processes with virtual photons give rise to logarithmically divergent integrals; to obtain a finite result, it is necessary to add the cross section for the emission of an additional photon with energy $\omega < \omega_{\min}$ which cancels this divergence. Usually $\omega_{\min}$ is identified with the finite energy resolution of the detector and assumed to be small compared to the mass of the radiating particle, such that the approximation of classical currents can be used. The contribution of harder photons indistinguishable from a single photon is then evaluated numerically according to the conditions of the experiment (see e.g. [13]). In the problem of muon propagation, however, the particle may traverse several kilometers of material before the energy losses can be seen by the detector. Therefore the cross section has to be integrated over all kinematically allowed states of the additional photon. So the energy loss depends only on the primary energy of the muon.

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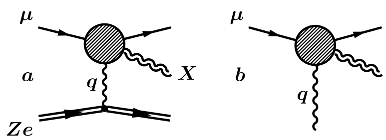

Fig. 1. (*a*) Symbolic Diagram of interaction of a muon with an atom. (*b*) Equivalent Diagram in the Weizsäcker–Williams method. The shaded blob denotes the internal part of the diagram. X denotes particles created in the collision.

Unless stated otherwise, all equations are presented in a system of units where $\hbar = c = m_\mu = 1$.

2. Method

The calculation reported here is based on the Weizsäcker–Williams method [14,15], which approximates the effect of a nucleus by a spectrum of equivalent photons. This method allows to express the bremsstrahlung cross section through the Compton cross section convolved with the equivalent photon flux. Using the radiative corrections to the Compton effect in [16], the radiative corrections to the bremsstrahlung spectrum were first calculated in the soft-photon approximation in [17] for an unscreened or totally screened nucleus.

2.1. Conventional Weizsäcker–Williams method

Considering the collision of a fast muon with an atom, we introduce two systems of reference: the laboratory system K_Z in which the atom is at rest and the muon has a Lorentz factor $\gamma \gg 1$, and the system K_μ in which the muon is at rest and the atom has a Lorentz factor of γ. The interaction of the muon with the atom can be described symbolically by the diagram in Fig. 1(*a*). The shaded blob denotes the internal part of the diagram, the double line X denotes particles created in the collision. Diagram Fig. 1(*b*) describes a similar process due to collision of a real photon with a muon. We consider two cases here:

1. $X = \gamma$. In this case (*a*) is bremsstrahlung and we assume that the blob also contains radiative corrections; (*b*) is Compton scattering with radiative corrections.
2. $X = 2\gamma$. In this case (*a*) is double bremsstrahlung and (*b*) is double Compton scattering.

In the Weizsäcker–Williams method the field of the atom in K_μ is replaced by a flux of equivalent photons with the spectrum $n(\omega)\,\mathrm{d}\omega$. This allows to relate the differential cross sections of the processes (*a*) and (*b*) by the relation

$$\mathrm{d}\sigma_a = \mathrm{d}\sigma_b n(\omega)\,\mathrm{d}\omega, \tag{2}$$

and for the total cross section

$$\int \mathrm{d}\sigma_a = \int \mathrm{d}\sigma_b n(\omega)\,\mathrm{d}\omega. \tag{3}$$

It is convenient to calculate the cross section σ_b in the frame K_μ. Since the cross section is Lorentz-invariant, the transition from K_μ to K_Z is trivial.

In K_Z the average energy loss per unit length caused by the process (*a*) is given by

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = NE\Sigma, \quad \Sigma = \frac{1}{E} \int (E - E')\,\mathrm{d}\sigma_a, \tag{4}$$

where E (E') is the initial (final) muon energy, N is the number density of target atoms per unit volume. The quantity Σ can be rewritten in a relativistically invariant form as

$$\Sigma = \frac{1}{(up)} \int ((up) - (up'))\,\mathrm{d}\sigma_a, \tag{5}$$

where u is the 4-velocity of the atom and p, p' are the initial and final 4-momenta of the muon respectively, $(up) = u^0 p^0 - \mathbf{up}$ is the scalar product of 4-vectors. Using (2) it is possible to rewrite this as

$$\Sigma = \int \frac{(up) - (up')}{(up)}\,\mathrm{d}\sigma_b n(\omega)\,\mathrm{d}\omega. \tag{6}$$

In this equation we will calculate the integrand in the K_μ frame. For Compton scattering the energy-momentum conservation gives $(u, p - p') = (u, q' - q)$, where q (q') is the initial (final) photon 4-momentum. In the frame K_μ we have $(uq)/\gamma = \omega(1 - \beta) \approx \omega/(2\gamma^2)$, where ω is the initial photon energy. Since $\omega \sim \gamma$, the ratio $(uq)/\gamma \sim 1/\gamma$ is negligible. The other term is

$$\frac{(uq')}{\gamma} = \omega'(1 - \beta\cos\theta) \approx \frac{1}{2}\omega'(\theta^2 + 1/\gamma^2). \tag{7}$$

In Compton scattering $\theta \lesssim 1/\sqrt{\gamma}$ [18] and the second term in parentheses is negligible. Therefore we obtain for the first case

$$1 - \frac{(up')}{(up)} \approx 1 - \frac{\omega'}{\omega}. \tag{8}$$

Similarly for double Compton scattering we have

$$1 - \frac{(up')}{(up)} \approx \omega_1(1 - \cos\theta_1) + \omega_2(1 - \cos\theta_2), \tag{9}$$

where $\omega_{1,2}, \theta_{1,2}$ are the energies and angles of the final photons.

2.2. Modified Weizsäcker–Williams method

For a point-like nucleus, the pseudophoton flux in the rest frame of the muon is given by [18]

$$n(\omega)\,\mathrm{d}\omega = \frac{2}{\pi} Z^2 \alpha \frac{\mathrm{d}\omega}{\omega} \ln\frac{\gamma}{\omega} \tag{10}$$

where γ is the Lorentz factor of the incident particle in the laboratory frame, or, equivalently the Lorentz factor of the nucleus in the rest frame of the muon.

For muons, it is necessary to take into account the extended nucleus and the screening of the nucleus by atomic electrons, because the characteristic momentum transfer of $q \sim m_\mu$ is comparable to the inverse radius of the nucleus and the minimum momentum transfer $\delta \sim m_\mu^2/E$ is comparable to or smaller than the inverse radius of the atom [3].

For an atom with nuclear and atomic formfactors $F_\mathrm{n}(q^2), F_\mathrm{a}(q^2)$ we obtain

$$n(\omega)\,\mathrm{d}\omega = \frac{\alpha Z^2\,\mathrm{d}\omega}{\pi\omega} \int\limits_{\omega^2/\gamma^2}^{\infty} \frac{\tau - \omega^2/\gamma^2}{\tau^2} \left(F_\mathrm{n}(\tau) - F_\mathrm{a}(\tau)\right)^2 \mathrm{d}\tau. \tag{11}$$

In this work the charge distribution of the nucleus and of the atomic electrons are described by a Gaussian and an exponential distribution, respectively, resulting in the form factors

$$F_\mathrm{n}(q^2) = \exp\left[-\frac{q^2 R_\mathrm{n}^2}{6}\right], \tag{12}$$

$$F_\mathrm{a}(q^2) = \left[1 + \frac{q^2 R_\mathrm{a}^2}{12}\right]^{-2} \tag{13}$$

with R the Rms-radius of the charge distribution. The atomic and nuclear radius can be parametrized for light and medium nuclei as [19]