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Baryon number violation and novel canonical anti-commutation relations

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ABSTRACT

The possible neutron–antineutron oscillation is described by an effective quadratic Lagrangian analogous to the BCS theory. It is shown that the conventional equal-time anti-commutation relations of the neutron variable $n(t,\vec{x})$ are modified by the baryon number violating terms. This is established by the Bjorken–Johnson–Low prescription and also by the canonical quantization combined with equations of motion. This novel canonical behavior can give rise to an important physical effect, which is illustrated by analyzing the Lagrangian that violates the baryon number but gives rise to the degenerate effective Majorana fermions and thus no neutron–antineutron oscillation. Technically, this model is neatly treated using a relativistic analogue of the Bogoliubov transformation.

1. Introduction

The possible neutron oscillation is analyzed by the quadratic effective Hermitian Lagrangian with general $\Delta B = 2$ terms added [1–12],

$$\begin{aligned}\mathcal{L}_0 &= \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - m\overline{n}(x)n(x) \\ &- \frac{1}{2}\epsilon_1 [e^{i\alpha} n^T(x) C n(x) + e^{-i\alpha}\overline{n}(x) C \overline{n}^T(x)] \\ &- \frac{1}{2}\epsilon_5 [n^T(x) C\gamma_5 n(x) - \overline{n}(x) C\gamma_5 \overline{n}^T(x)], \end{aligned} \quad (1)$$

where m, ϵ_1, ϵ_5 and α are real parameters. The most general quadratic Hermitian Lagrangian is written in the form (1) using the phase freedom of $n(x) \to n(x) = e^{i\beta} n'(x)$; under this change of naming the field, the physical quantities in (1) such as mass eigenvalues are obviously invariant. But C (and thus CP) transformation rules of the solution of the Lagrangian (1) are modified. In the present paper, we adopt the above phase convention which is different from the one used in [13].

The first $\Delta B = 2$ term with real ϵ_1 breaks the γ^0-parity which is defined by

$$n(t,\vec{x}) \to \gamma^0 n(t,-\vec{x}), \quad n^c(t,\vec{x}) \to -\gamma^0 n^c(t,-\vec{x}) \quad (2)$$

with $n^c(t,\vec{x}) \equiv C\overline{n(t,\vec{x})}^T$, while the second term with real ϵ_5 preserves γ^0-parity. In contrast, the first term with real ϵ_1 preserves $i\gamma^0$-parity which is defined by

$$n(t,\vec{x}) \to i\gamma^0 n(t,-\vec{x}), \quad n^c(t,\vec{x}) \to i\gamma^0 n^c(t,-\vec{x}), \quad (3)$$

while the second term with real ϵ_5 breaks $i\gamma^0$-parity. The $i\gamma^0$-parity is natural in the analysis of the Majorana fermion since it preserves the reality of the field in the Majorana representation. In the discussion of discrete symmetries of the general effective Lagrangian (1), one is bound to adopt the $i\gamma^0$-parity, and the CP defined in terms of $i\gamma^0$-parity is broken only when $\alpha \neq 0$ in (1). Our notational conventions follow [14], in particular, $C = i\gamma^2\gamma^0$.

The model (1) has been studied by various authors in the past [1–12]. We have given an exact solution of (1) with $\alpha \neq 0$ and showed that the neutron oscillation cannot detect the effect of CP violation, although the absolute rate of the oscillation is influenced by $\alpha \neq 0$ [13]. We have also shown that the choice $\epsilon_1 = 0$ gives rise to the degenerate effective Majorana masses and thus no oscillation. Nevertheless, physically the effect of γ_0-parity preserving $\Delta B = 2$ terms is not negligible [13], and it may appear in the instability of nuclei. This effect is related to the interesting novel anti-commutation relations of neutron variables such as $\{n(t,\vec{x}), n(t,\vec{y})\} = 0$ but $\{\dot{n}(t,\vec{x}), n(t,\vec{y})\} \neq 0$, which is analyzed in detail in the present paper. This effect is specific to the baryon number violating theory.

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For example, in a model analogous to the Nambu–Jona-Lasinio model [15] such as

$$\mathcal{L} = \bar{n}(x)i\gamma^{\mu}\partial_{\mu}n(x) - m\bar{n}(x)n(x) - \lambda\bar{n}(x)(1+\gamma_5)n(x)\bar{n}(x)(1-\gamma_5)n(x), \tag{4}$$

where the baryon number is strictly conserved and thus

$$\langle T^{\star}n(x)n(y)\rangle = 0, \tag{5}$$

the above mentioned novel behavior of the canonical anti-commutation relations does not appear. Eq. (5) is a consequence of the Ward–Takahashi identity for the baryon number conservation in (4). Alternatively, one may define the baryon number operator $B = \int d^3x\bar{n}(x)\gamma^0 n(x)$, which is time-independent $\frac{d}{dt}B = 0$, and $\exp[-i\alpha B]n(x)\exp[i\alpha B] = e^{i\alpha}n(x)$. The baryon number conserving vacuum implies $\exp[i\alpha B]|0\rangle = |0\rangle$, and thus

$$\begin{aligned}\langle 0|T^{\star}n(x)n(y)|0\rangle &= \langle 0|\exp[-i\alpha B]T^{\star}n(x)\exp[i\alpha B] \\ &\quad\times\exp[-i\alpha B]n(y)\exp[i\alpha B]|0\rangle \\ &= e^{2i\alpha}\langle 0|T^{\star}n(x)n(y)|0\rangle,\end{aligned} \tag{6}$$

for arbitrary α. We thus conclude (5).

2. Degenerate Majorana masses

We have shown that the effective Lagrangian (1) with $\epsilon_1 = 0$, i.e.

$$\mathcal{L} = \bar{n}(x)i\gamma^{\mu}\partial_{\mu}n(x) - m\bar{n}(x)n(x) - \frac{1}{2}\epsilon_5[n^T(x)C\gamma_5 n(x) - \bar{n}(x)C\gamma_5\bar{n}^T(x)], \tag{7}$$

which is invariant under the "γ^0-parity", gives rise to the degenerate Majorana fermions [13], even in a more general context, as is discussed later. The degeneracy of Majorana masses implies the absence of the conventional neutron oscillation despite the presence of the ϵ_5-term with $\Delta B = 2$.

We use the Lagrangian in (7) to analyze the novel anti-commutation relations. To solve (7), we apply an analogue of Bogoliubov transformation, $(n, n^c) \to (N, N^c)$, defined as [13]

$$\begin{pmatrix} N(x) \\ N^c(x)\end{pmatrix} = \begin{pmatrix}\cos\phi\, n(x) - \gamma_5\sin\phi\, n^c(x) \\ \cos\phi\, n^c(x) + \gamma_5\sin\phi\, n(x)\end{pmatrix}, \tag{8}$$

with

$$\sin 2\phi = \epsilon_5/\sqrt{m^2 + (\epsilon_5)^2}. \tag{9}$$

One can confirm the classical consistency condition $N^c = C\overline{N}^T(x)$ using the expressions of the right-hand side of (8). One can also confirm

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\{\bar{N}i\partial\!\!\!/ N + \bar{N}^c i\partial\!\!\!/ N^c\} \\ &= \frac{1}{2}\{\bar{n}i\partial\!\!\!/ n + \bar{n^c}i\partial\!\!\!/ n^c\}.\end{aligned} \tag{10}$$

We can then show that the anticommutators are preserved, i.e.,

$$\begin{aligned}&\{N(t,\vec{x}), N^c(t,\vec{y})\} = \{n(t,\vec{x}), n^c(t,\vec{y})\}, \\ &\{N_{\alpha}(t,\vec{x}), N_{\beta}(t,\vec{y})\} = \{N^c_{\alpha}(t,\vec{x}), N^c_{\beta}(t,\vec{y})\} = 0,\end{aligned} \tag{11}$$

and thus the condition of a canonical transformation required for the Bogoliubov transformation is satisfied. This condition of the canonical transformation is valid irrespective of the mass values of n and N. A transformation analogous to (8) has been successfully used in the analysis of neutrino masses in the seesaw mechanism [16,17].

After the Bogoliubov transformation, (7) becomes

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[\overline{N}(x)\,(i\partial\!\!\!/ - M)\,N(x) + \overline{N^c}(x)\,(i\partial\!\!\!/ - M)\,N^c(x)\right] \\ &= \frac{1}{2}\left[\overline{\psi_+}(x)\,(i\partial\!\!\!/ - M)\,\psi_+(x) + \overline{\psi_-}(x)\,(i\partial\!\!\!/ - M)\,\psi_-(x)\right],\end{aligned} \tag{12}$$

where the Majorana fermions are defined by

$$\psi_{\pm}(x) = \frac{1}{\sqrt{2}}[N(x) \pm N^c(x)] \tag{13}$$

which satisfy

$$\psi^c_+(x) = \psi_+(x), \quad \psi^c_-(x) = -\psi_-(x). \tag{14}$$

The mass parameter is defined by

$$M \equiv \sqrt{m^2 + (\epsilon_5)^2}. \tag{15}$$

This implies that the Bogoliubov transformation maps the original theory to a theory of quasiparticles described by the field $N(x)$, characterized by a new mass M (ϵ_5 corresponds to the energy gap). The Bogoliubov transformation maps a linear combination of a Dirac fermion and its charge conjugate to another Dirac fermion and its charge conjugate, and thus the Fock vacuum is mapped to a new vacuum defined by $\mathcal{L}$ at $t = 0$ (see, for example, [17]). It is important that the Bogoliubov transformation (8) preserves the CP symmetry, although it does not preserve the transformation properties under $i\gamma^0$-parity and C separately.

The solution of the starting Lagrangian (7) is written as,

$$\begin{pmatrix} n(x) \\ n^c(x)\end{pmatrix} = \begin{pmatrix}\cos\phi N(x) + \gamma_5\sin\phi N^c(x) \\ \cos\phi N^c(x) - \gamma_5\sin\phi N(x)\end{pmatrix}, \tag{16}$$

with $\sin 2\phi$ defined in (9). The solution can also be expressed in terms of Majorana fermions defined in (13) using

$$\begin{aligned}N(x) &= [\psi_+(x) + \psi_-(x)]/\sqrt{2} \\ N^c(x) &= [\psi_+(x) - \psi_-(x)]/\sqrt{2}.\end{aligned} \tag{17}$$

When one generates the neutron experimentally, one obtains the field expressed as

$$\begin{aligned}n(x) &= \cos\phi N(x) + \gamma_5\sin\phi N^c(x) \\ &= \frac{1}{\sqrt{2}}\{\cos\phi[\psi_+(x) + \psi_-(x)] + \gamma_5\sin\phi[\psi_+(x) - \psi_-(x)]\}, \\ n^c(x) &= \cos\phi N^c(x) - \gamma_5\sin\phi N(x) \\ &= \frac{1}{\sqrt{2}}\{\cos\phi[\psi_+(x) - \psi_-(x)] - \gamma_5\sin\phi[\psi_+(x) + \psi_-(x)]\},\end{aligned} \tag{18}$$

but no oscillation in the conventional sense

$$n(x) \to n^c(x) \to n(x) \to \ldots., \tag{19}$$

takes place due to the mass degeneracy of the Majorana fermions $\psi_{\pm}(x)$. Note that the neutron–antineutron oscillation $n(x) \to n^c(x)$ occurs due to the mass differences of the two Majorana particles appearing in the expressions of $n(x)$ and $n^c(x)$. It may thus seem that no physical effects of the baryon number violation such as the decay originating from $n(x)$ into two distinct final states appear.

However, $n(x)$ and $n^c(x)$ are not orthogonal, in the sense

$$\langle T^{\star}n^c(x)\bar{n}(y)\rangle = \int\frac{d^4p}{(2\pi)^4}\frac{(-i)\gamma_5 M\sin 2\phi}{p^2 - M^2 + i\epsilon}e^{-ip(x-y)}, \tag{20}$$