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A two-dimensional soliton system of vortex and Q-ball

A.Yu. Loginov

Tomsk Polytechnic University, 634050 Tomsk, Russia

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ABSTRACT

The $(2+1)$-dimensional gauge model describing two complex scalar fields that interact through a common Abelian gauge field is considered. It is shown that the model has a soliton solution that describes a system consisting of a vortex and a Q-ball. This two-dimensional system is electrically neutral, nevertheless it possesses a nonzero electric field. Moreover, the soliton system has a quantized magnetic flux and a nonzero angular momentum. Properties of this vortex-Q-ball system are investigated by analytical and numerical methods. It is found that the system combines properties of topological and nontopological solitons.

1. Introduction

Topological solitons of $(2+1)$-dimensional field models play an important role in field theory, physics of condensed state, cosmology, and hydrodynamics. First of all, it is necessary to mention vortices of the effective theory of superconductivity [1] and vortices of the $(2+1)$-dimensional Abelian Higgs model [2]. Another important example is given by the soliton solution of the $(2+1)$-dimensional nonlinear $O(3)$ σ model [3] that effectively describes the behavior of a ferromagnet in the critical region.

Two-dimensional soliton solutions of Abelian Maxwell gauge models are necessarily electrically neutral. This is because the $(2+1)$-dimensional Maxwell electrodynamics does not admit the existence of electrically charged spatially localized solutions with finite energy [4], in contrast to the $(3+1)$-dimensional case. However, the electrical neutrality does not forbid the existence of two-dimensional solitons possessing an electric field.

In this Letter we consider a two-dimensional soliton system consisting of an Abelian vortex and a Q-ball. The vortex and the Q-ball interact through a common Abelian gauge field. This electrically neutral soliton system possesses a radial electric field, carries a quantized magnetic flux, and has a nonzero angular momentum. The soliton system combines the properties of vortex and Q-ball. The interaction between the vortex and the Q-ball by means of a common gauge field leads to a significant change of their shapes.

E-mail address: aloginov@tpu.ru.

2. Lagrangian and field equations of the model

The $(2+1)$-dimensional model we are interested in is described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(D_{\mu}\phi\right)^{*} D^{\mu}\phi - V\left(|\phi|\right) + \left(D_{\mu}\chi\right)^{*} D^{\mu}\chi - U\left(|\chi|\right), \tag{1}$$

where ϕ and χ are complex scalar fields that are minimally coupled to the Abelian gauge field A_{μ} through covariant derivatives:

$$D_{\mu}\phi = \partial_{\mu}\phi - ieA_{\mu}\phi, \quad D_{\mu}\chi = \partial_{\mu}\chi - iqA_{\mu}\chi. \tag{2}$$

The self-interaction potentials $V\left(|\phi|\right)$ and $U\left(|\chi|\right)$ are

$$V\left(|\phi|\right) = \frac{\lambda}{2}\left(\phi^{*}\phi - v^{2}\right)^{2},$$
$$U\left(|\chi|\right) = m^{2}\chi^{*}\chi - g\left(\chi^{*}\chi\right)^{2} + h\left(\chi^{*}\chi\right)^{3}, \tag{3}$$

where λ, g, and h are the positive self-interaction constants, m is the mass of the scalar χ-particle, and v is the vacuum average of the complex scalar field ϕ. We suppose that the potential $U\left(|\chi|\right)$ has the global minimum at $\chi = 0$ and a local one at some $|\chi| \neq 0$; hence we have the following condition for the parameters m, g, and h:

$$\frac{g^{2}}{4m^{2}} < h < \frac{g^{2}}{3m^{2}}. \tag{4}$$

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Note that if the coupling constant q in Eq. (2) is set equal to zero, then model (1) has the soliton solution describing an Abelian vortex and a two-dimensional Q-ball. However, there is no electric field in this case, so the vortex and the Q-ball do not interact with each other.

The Lagrangian (1) is invariant under the local gauge transformations:

$$\begin{aligned}\phi(x) &\to \phi'(x) = \exp(ie\Lambda(x))\,\phi(x),\\ \chi(x) &\to \chi'(x) = \exp(iq\Lambda(x))\,\chi(x),\\ A_\mu(x) &\to A'_\mu(x) = A_\mu(x) + \partial_\mu\Lambda(x).\end{aligned} \tag{5}$$

Moreover, the Lagrangian (1) is also invariant under the two independent global gauge transformations:

$$\begin{aligned}\phi(x) &\to \phi'(x) = \exp(i\alpha)\,\phi(x),\\ \chi(x) &\to \chi'(x) = \exp(i\beta)\,\chi(x).\end{aligned} \tag{6}$$

The corresponding Noether currents are

$$\begin{aligned}j^\mu_\phi &= -i\left[\phi^* D^\mu\phi - (D^\mu\phi)^*\phi\right],\\ j^\mu_\chi &= -i\left[\chi^* D^\mu\chi - (D^\mu\chi)^*\chi\right].\end{aligned} \tag{7}$$

By varying the action $S = \int \mathcal{L} d^3x$ in A_μ, ϕ^*, and χ^*, we obtain the field equations of the model:

$$\partial_\mu F^{\mu\nu} = j^\nu, \tag{8}$$

$$D_\mu D^\mu\phi + \lambda\left(\phi^*\phi - v^2\right)\phi = 0, \tag{9}$$

$$D_\mu D^\mu\chi + \left(m^2 - 2g(\chi^*\chi) + 3h(\chi^*\chi)^2\right)\chi = 0, \tag{10}$$

where the electromagnetic current j^μ is expressed in terms of Noether currents:

$$j^\mu = ej^\mu_\phi + qj^\mu_\chi. \tag{11}$$

Using the well-known formula $T_{\mu\nu} = 2\partial\mathcal{L}/\partial g^{\mu\nu} - g^{\mu\nu}\mathcal{L}$, we obtain the symmetric energy-momentum tensor of the model

$$\begin{aligned}T_{\mu\nu} = &- F_{\mu\lambda}F_\nu^{\ \lambda} + \frac{1}{4}g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho}\\ &+ (D_\mu\phi)^* D_\nu\phi + (D_\nu\phi)^* D_\mu\phi\\ &- g_{\mu\nu}\left((D_\mu\phi)^* D^\mu\phi - V(|\phi|)\right)\\ &+ (D_\mu\chi)^* D_\nu\chi + (D_\nu\chi)^* D_\mu\chi\\ &- g_{\mu\nu}\left((D_\mu\chi)^* D^\mu\chi - U(|\chi|)\right).\end{aligned} \tag{12}$$

In particular, the energy density can be written as

$$\begin{aligned}T_{00} = &\frac{1}{2}E_iE_i + \frac{1}{2}B^2\\ &+ (D_0\phi)^* D_0\phi + (D_i\phi)^* D_i\phi + V(|\phi|)\\ &+ (D_0\chi)^* D_0\chi + (D_i\chi)^* D_i\chi + U(|\chi|),\end{aligned} \tag{13}$$

where $E_i = F_{0i}$ are the components of electric field strength and $B = -F_{12}$ is the magnetic field strength.

Let us fix the gauge as follows: $\partial_0\phi = 0$. We want to find a soliton solution of model (1) that minimizes the energy functional $E = \int T_{00} d^2x$ at the fixed value of the Noether charge $Q_\chi = \int j^0_\chi d^2x$. From the method of Lagrange multipliers it follows that the soliton is an unconditional extremum of the functional

$$F = \int \mathcal{H} d^2x - \omega\int j^0_\chi d^2x = E - \omega Q_\chi, \tag{14}$$

where $\mathcal{H}$ is the Hamiltonian density and ω is the Lagrange multiplier. Let us write the Noether charge Q_χ in terms of the canonically conjugated variables:

$$Q_\chi = i\int\left(\chi\pi_\chi - \chi^*\pi_{\chi^*}\right)d^2x, \tag{15}$$

where $\pi_\chi = \partial\mathcal{L}/\partial(\partial_0\chi) = (D_0\chi)^*$ and $\pi_{\chi^*} = \partial\mathcal{L}/\partial(\partial_0\chi^*) = D_0\chi$ are the generalized momenta canonically conjugated to χ and χ^*, respectively.

The extremum condition for the functional F is written as

$$\delta F = \delta H - \omega\delta Q_\chi = 0, \tag{16}$$

where a variation of the Noether charge Q_χ is written in terms of the canonically conjugate variables:

$$\delta Q_\chi = i\int\left(\chi\delta\pi_\chi + \pi_\chi\delta\chi - \text{c.c.}\right)d^2x. \tag{17}$$

Using the Hamilton field equations and Eqs. (16) and (17), we obtain:

$$\partial_0\chi = \frac{\delta H}{\delta\pi_\chi} = i\omega\chi, \quad \partial_0\chi^* = \frac{\delta H}{\delta\pi_{\chi^*}} = -i\omega\chi^*, \tag{18}$$

while time derivatives of the other model's fields are equal to zero. From Eq. (18) we get the time dependence of the scalar field χ

$$\chi(x) = \chi(\mathbf{x})\exp(i\omega t), \tag{19}$$

whereas the other fields of the model do not depend on time in the gauge $\partial_0\phi = 0$. From Eq. (16) it follows that the important relation holds for the soliton solution:

$$\frac{dE}{dQ_\chi} = \omega, \tag{20}$$

where ω is some function of Q_χ.

3. The ansatz and some properties of the solution

To find the soliton solution of field equations (8), (9), and (10), we use the following ansatz for the model's fields:

$$\begin{aligned}A^\mu(x) &= \left(\frac{A_0(r)}{er}, \frac{1}{er}\epsilon_{ij}n_jA(r)\right),\\ \phi(x) &= v\exp(-iN\theta)\,F(r),\\ \chi(x) &= \sigma(r)\exp(i\omega t),\end{aligned} \tag{21}$$

where ϵ_{ij} and n_j are the components of the two-dimensional antisymmetric tensor ($\epsilon_{12} = 1$) and the radial unit vector $\mathbf{n} = (\cos(\theta), \sin(\theta))$, respectively. Note that the fields A^i and ϕ are described by the vortex ansatz that was used in [5], while the scalar field χ is described by the Q-ball ansatz [6]. Note also that ansatz (21) completely fixes the model's gauge.

Substituting ansatz (21) into field equations (8), (9), and (10), we obtain the system of ordinary differential equations for the ansatz functions $A_0(r)$, $A(r)$, $F(r)$, and $\sigma(r)$:

$$\begin{aligned}&A_0''(r) - \frac{A_0'(r)}{r} + \frac{A_0(r)}{r^2}\\ &-\left(2e^2v^2F(r)^2 + 2q^2\sigma(r)^2\right)\\ &\times A_0(r) + 2eq\omega r\sigma(r)^2 = 0,\end{aligned} \tag{22}$$

$$\begin{aligned}&A''(r) - \frac{A'(r)}{r} - 2e^2v^2(N + A(r))\\ &\times F(r)^2 - 2q^2\sigma(r)^2A(r) = 0,\end{aligned} \tag{23}$$