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Manipulating lightcone fluctuations in an analogue cosmic string

Jiawei Hu [a], Hongwei Yu [a,b,*]

[a] Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha, Hunan 410081, China
[b] Center for Nonlinear Science and Department of Physics, Ningbo University, Ningbo, Zhejiang 315211, China

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ABSTRACT

We study the flight time fluctuations in an anisotropic medium inspired by a cosmic string with an effective fluctuating refractive index caused by fluctuating vacuum electric fields, which are analogous to the lightcone fluctuations due to fluctuating spacetime metric when gravity is quantized. The medium can be realized as a metamaterial that mimics a cosmic string in the sense of transformation optics. For a probe light close to the analogue string, the flight time variance is ν times that in a normal homogeneous and isotropic medium, where ν is a parameter characterizing the deficit angle of the spacetime of a cosmic string. The parameter ν, which is always greater than unity for a real cosmic string, is determined by the dielectric properties of the metamaterial for an analogue string. Therefore, the flight time fluctuations of a probe light can be manipulated by changing the electric permittivity and magnetic permeability of the analogue medium. We argue that it seems possible to fabricate a metamaterial that mimics a cosmic string with a large ν in laboratory so that a currently observable flight time variance might be achieved.

1. Introduction

Quantization of fundamental interactions such as electromagnetic, weak and strong interactions, has achieved great success, on one hand, but on the other hand, quantization of gravity, the fundamental interaction known to the mankind the earliest, still remains elusive. If, however, we accept that the basic quantum principles we are already familiar with apply as well to a quantum theory of gravity, we can make some predictions about expected quantum effects, even in the absence of a fundamental underlying theory. The uncertainty principle is one of such principles and one generic prediction arising from its application to the theory of gravity is that lightcones, the boundaries between spacelike and timelike regions, are no longer fixed, but smeared out due to the quantum fluctuations of spacetime metric. Since the ultraviolet divergences in quantum field theory arise from the lightcone singularities of two-point functions, it was even conjectured first by Pauli [1] and later investigated by other authors [2–5] that the divergences might be removed when gravity is quantized. The theoretical implications and the detectability of metric fluctuations have been extensively studied [6–20]. A direct result of lightcone fluctuations is that the flight time of a probe light signal from its source to a detector spreads about the classical value in both directions [14–20]. These flight time fluctuations are in principle observable but are extremely small in general and seem to be undetectable in experiment in the foreseeable future.

Therefore, it is of interest to resort to analogue systems to see whether some basic predictions of quantum gravity can be analogously verified there. Indeed, if some such predictions can be tested, we may be able to derive useful constraints on the properties of the true underlying theory in which gravity is quantized. Based on nonlinear optics, Ford et al. proposed an analogue model for quantum lightcone fluctuations [21,22]. In a nonlinear medium, the flight time of a probe light fluctuates due to a fluctuating effective refractive index when the medium is subjected to a fluctuating background field, which is analogous to the lightcone fluctuation when gravity is quantized. The fluctuating background field can be either a single field mode in a squeezed state [21], or a bath of multi-mode fluctuating electromagnetic fields in vacuum [22]. These are analogue models for active gravitational field fluctuations, which are fluctuations of the dynamical degrees of freedom of gravity itself. In Refs. [23,24], an analogue model for passive fluctuations of gravity driven by quantum stress tensor fluctuations has also been proposed.

* Corresponding author at: Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha, Hunan 410081, China.
E-mail address: hwyu@hunnu.edu.cn (H. Yu).

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In this paper, we study the flight time fluctuation of certain probe light pulses that arises from a fluctuating refractive index due to electromagnetic vacuum fluctuations in an anisotropic medium that mimics a cosmic string in terms of the classical propagation of light. Such a medium can be realized as a metamaterial in experiment. In particular, we demonstrate that the flight time fluctuations can be amplified compared with those in a normal medium by manipulating the electric permittivity and magnetic permeability of the analogue medium, and remarkably, a currently experimentally observable flight time variance would be obtained if an analogue cosmic string with a large enough ν could be fabricated in laboratory. Here let us note that the lightcone fluctuations due to metric fluctuations in the cosmic string spacetime have recently been studied in Ref. [20]. The Lorentz–Heaviside units with $\hbar = c = 1$ are used in this paper unless specified.

2. The basic formalism

In a nonlinear medium, the electric polarization P_i can be expanded in a power series of the electric field E_i as

$$P_i = P_i^{(1)} + P_i^{(2)} + \cdots = \chi_{ij}^{(1)} E^j + \chi_{ijk}^{(2)} E^j E^k + \cdots \,. \tag{1}$$

Here $\chi^{(i)}$ is the i-th order susceptibility tensor. We write the total electric field E^i as a sum of a background field $E_0^i(\omega_0)$ and a probe field $E_1^i(\omega_1)$. So, the second order polarization $P_i^{(2)}$ takes the form

$$P_i^{(2)}(\omega_m + \omega_n) = \sum_{m,n=0}^{1} \chi_{ijk}^{(2)}(\omega_m + \omega_n) E_m^j(\omega_m) E_n^k(\omega_n) \,. \tag{2}$$

We assume that for the background field $E_0^i(\omega_0)$, the second order susceptibility tensor $\chi_{ijk}^{(2)}(2\omega_0)$ can be neglected. So, in the absence of the probe field E_1^i, the medium the background field E_0^i propagates in can be treated as a linear medium characterized by a dielectric tensor ε_{ij} such that $D^i = \varepsilon^{ij} E_j$ [22]. Here we have neglected dispersion in the frequency range of the background fields. On the other hand, the magnetization is also assumed to be linear with the applied magnetic field, so $B^i = \mu^{ij} H_j$. In this paper, we are interested in an anisotropic metamaterial medium in which the propagation of light rays is equivalent to that in a static, straight cosmic string spacetime, with the line element being

$$ds^2 = dt^2 - d\rho^2 - \frac{\rho^2}{\nu^2} d\phi^2 - dz^2 \,. \tag{3}$$

Here $\nu = (1 - 4G\mu)^{-1}$, with μ the mass per unit length of the string, and G the Newtonian constant of gravitation. Let $\theta = \phi/\nu$, the line element can be rewritten as

$$ds^2 = dt^2 - d\rho^2 - \rho^2 d\theta^2 - dz^2 \,, \tag{4}$$

where $\theta \in [0, 2\pi/\nu)$. This metric describes a flat spacetime with a deficit angle $8\pi G\mu$. In the framework of transformation optics [25–27], the electric permittivity and magnetic permeability tensors of a medium that mimics the propagation of light in the cosmic string spacetime take the form

$$\varepsilon^{ij} = \mu^{ij} = n_B \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{\nu^2}{\rho^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{5}$$

in the cylindrical coordinate. After a coordinate transformation from the cylindrical coordinate to the Cartesian coordinate, ε^{ij} and μ^{ij} become

$$\varepsilon^{ij} = \mu^{ij} = n_B \begin{pmatrix} \cos\phi^2 + \nu^2 \sin\phi^2 & (1-\nu^2)\cos\phi\sin\phi & 0 \\ (1-\nu^2)\cos\phi\sin\phi & \sin\phi^2 + \nu^2\cos\phi^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{6}$$

which is in agreement with those obtained in Ref. [28] for spinning cosmic strings when the angular momentum of the cosmic string approaches zero, as expected. Note that when $\nu = 1$, the above tensor describes a normal homogeneous and isotropic medium and empty space if the refractive index of the background field n_B is further set to be unity.

The electromagnetic wave equation in such a medium can be written as

$$-\frac{1}{\sqrt{\gamma}} \partial_j \sqrt{\gamma} \gamma^{jk} \partial_k \gamma^{im} E_m + \frac{1}{\sqrt{\gamma}} \partial_j \sqrt{\gamma} \gamma^{ik} \partial_k \gamma^{jm} E_m + \frac{1}{v_B^2} \frac{\partial^2 \gamma^{im} E_m}{\partial t^2} = 0 \,, \tag{7}$$

where $\gamma = \rho/\nu$ is the determinant of the spatial metric tensor of the cosmic string spacetime $\gamma_{ij} = \mathrm{diag}(1, \rho^2/\nu^2, 1)$. This equation takes the same form as that in the cosmic string spacetime with an effective speed of light $v_B = 1/n_B$.

Now we consider a probe light E_1^i, which is much smaller than the background field E_0^i, while its frequency ω_1 is much larger than that of the background field ω_0 [21,22]. If the probe field is propagating in the z-direction and is polarized in the ρ-direction, i.e. $E_1^i = \delta^{i\rho} E_1(t, z)$, the wave equation for E_1 takes the form

$$-\frac{\partial^2 E_1}{\partial z^2} + \frac{1}{v_P^2} \left[1 + \frac{1}{n_P^2} \left(\chi_{\rho\rho j}^{(2)} + \chi_{\rho j \rho}^{(2)} \right) E_0^j \right] \frac{\partial^2 E_1}{\partial t^2} = 0 \,, \tag{8}$$

where $n_P = 1/v_P$. This equation describes a wave propagating with a space and time dependent phase velocity

$$v \approx v_P \left[1 - \frac{1}{2n_P^2} \left(\chi_{\rho\rho j}^{(2)} + \chi_{\rho j \rho}^{(2)} \right) E_0^j \right] , \tag{9}$$

where $\left| \frac{1}{2n_P^2} \left(\chi_{\rho\rho j}^{(2)} + \chi_{\rho j \rho}^{(2)} \right) E_0^j \right| \ll 1$ is assumed. Generally, v_P is different from v_B due to dispersion. Here let us note that the unique properties of metamaterials are usually restricted to a narrow frequency range. However, in the derivation of Eq. (8), no assumption is made that the medium simulates the cosmic string spacetime in the frequency regime of the probe light.

3. Lightcone fluctuations in an analogue cosmic string

Consider that the probe light propagates through the metamaterial of an analogue cosmic string located at the z-axis, in the parallel direction from (t_1, ρ, ϕ, z_1) to (t_2, ρ, ϕ, z_2), with $d = z_2 - z_1$ the thickness of the material, as sketched in Fig. 1. The flight time of the probe signal through the material can then be expressed as

$$t = \int_{z_1}^{z_2} \frac{dz}{v} = n_P \int_{z_1}^{z_2} \left[1 + \frac{1}{2n_P^2} \left(\chi_{\rho\rho j}^{(2)} + \chi_{\rho j \rho}^{(2)} \right) E_0^j(t, \vec{x}) \right] dz \,. \tag{10}$$

This integration is along the path of the probe pulse, i.e. $z = v_P t = t/n_P$. In the present paper, we assume that the background field E_0 is the fluctuating vacuum electromagnetic field in the analogue medium of a cosmic string. The fluctuations of E_0 will cause the fluctuations of the flight time t. The relative flight time variance takes the form [21,22]