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Entropy/information flux in Hawking radiation

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ABSTRACT

Blackbody radiation contains (on average) an entropy of 3.9 ± 2.5 bits per photon. If the emission process is unitary, then this entropy is exactly compensated by "hidden information" in the correlations. We extend this argument to the Hawking radiation from GR black holes, demonstrating that the assumption of unitarity leads to a perfectly reasonable entropy/information budget. The key technical aspect of our calculation is a variant of the "average subsystem" approach developed by Page, which we extend beyond bipartite pure systems, to a tripartite pure system that considers the influence of the environment.

1. Introduction

The "information puzzle" due to the Hawking evaporation of GR black holes continues to provoke much heated discussion and debate [1–18]. On the other hand, there simply is no "information puzzle" associated with chemical burning [19], nor with the Hawking radiation from *analogue* black holes [20–41], where physics is manifestly unitary. (Horizons, if present at all, are *apparent/trapping* horizons; definitely not *event* horizons [42–44].) Previously we carefully analyzed the blackbody radiation from a "blackbody furnace" [19]. In the current article we focus on Hawking radiation from both *analogue* and GR black holes. Despite many claims to the contrary, (assuming unitarity and complete evaporation), the Hawking evaporation process is relatively benign, no worse than burning a lump of coal.

2. Entropy/information in blackbody radiation

When burning a lump of coal (or an encyclopaedia for that matter) in a blackbody furnace, individual photons in the resulting blackbody radiation carry (on average) an entropy/ information content of [19]

$$\langle \hat{S}_2 \rangle \approx 3.90 \pm 2.52 \text{ bits/photon}. \tag{1}$$

We use S to denote the physical entropy, $\hat{S} = S/k_B$ for the dimensionless entropy measured in nats (natural units), and $\hat{S}_2 = \hat{S}/\ln 2$. We now apply these results within the context of Hawking radiation, paying particular attention to the von Neumann entanglement entropy, and thence to the Page curve, as one of the main features underlying the firewall argument.

3. Hawking evaporation: analogue and GR black holes

Analogue black holes [20–29] have unitary Hawking flux; the relevant *blocking/acoustic* horizons are *apparent/trapping* horizons. Hawking quanta simply deliver a coarse-grained thermodynamic entropy [45] $S = \hbar\omega/T$ to the radiation field; exactly compensated by the information hidden in the correlations between the quanta [19]. *Analogue* black holes provide the only *experimental* evidence for the reality of Hawking radiation [30–36], showing a quite standard unitary preserving quantum physics without involving any "information puzzle".

If there is any "information puzzle" for GR black holes, it is not Hawking radiation *per se* that is the central issue. It is the assumed existence of *event* horizons (which certainly do exist in the classical limit) surviving in the semiclassical quantum realm that is the source of the potential difficulties.

The "information puzzle" can be traced back to Hawking's 1976 article [1] where he introduced the concept of "hidden surface", which was to be understood as a synonym for "absolute causal horizon". (Hawking has since twice abjured the semiclassical survival of event horizons [46,47].) Also note that *event* horizons are

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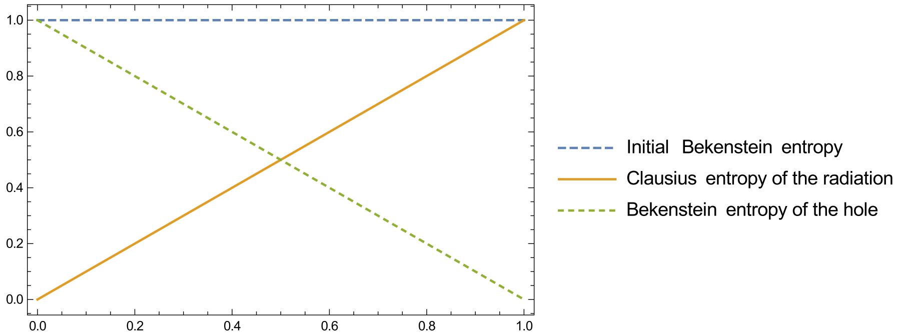

Fig. 1. Clausius (thermodynamic) entropy balance: As the black hole Bekenstein entropy (defined in terms of the area of the horizon) decreases the Clausius entropy of the radiation increases to keep total entropy constant and equal to the initial Bekenstein entropy.

simply not physically observable (in any finite size laboratory), whereas *apparent/trapping* horizons certainly are physically observable, at least in spherical symmetry [48]. Furthermore, even in a general relativity context, *event* horizons are simply not essential for generating a Hawking-like flux [49–52].

4. Thermodynamic entropy: Hawking flux from a GR black hole

For the Hawking evaporation of a GR black hole, we shall argue that classical thermodynamic entropy fluxes stay the same. Quantum entanglement entropy fluxes *might* in principle differ; that is essentially what all the arguing is about. To clarify these issues we shall compare and contrast the behaviour of the classical thermodynamic (Clausius) entropy with the quantum entanglement (von Neumann) entropy (of suitably defined subsystems).

4.1. Loss of Bekenstein entropy of the GR black hole

Let us first estimate the Bekenstein entropy loss of the black hole per emitted quanta. We assume for simplicity an exact Planck spectrum at the Hawking temperature, this being a good zeroth-order approximation to the actual physics [53,54]. For a Schwarzschild black hole we have

$$\frac{\mathrm{d}S}{\mathrm{d}N} = \frac{\mathrm{d}S/\mathrm{d}t}{\mathrm{d}N/\mathrm{d}t} = (8\pi k_B GM/\hbar c)(\hbar\langle\omega\rangle/c^2), \tag{2}$$

where so far we have only used the definition of Bekenstein entropy and the conservation of energy. Thus, for a Planck spectrum of emitted particles [19]

$$\frac{\mathrm{d}S}{\mathrm{d}N} = \frac{k_B\pi^4}{30\,\zeta(3)}. \tag{3}$$

This is Bekenstein entropy loss of the black hole (per emitted massless boson).

4.2. Gain of thermodynamic entropy of the radiation

Contrast this with the thermodynamic entropy gain (Clausius entropy gain) of the external radiation field (the Hawking flux) per emitted quanta. We have

$$\frac{\mathrm{d}S}{\mathrm{d}N} = \frac{\mathrm{d}E/T_H}{\mathrm{d}N} = \frac{\hbar\langle\omega\rangle}{T_H} = \frac{k_B\pi^4}{30\,\zeta(3)}. \tag{4}$$

Independent of the details of the microphysics, at the macroscopic level the Hawking radiation is essentially just (adiabatically) transferring the Bekenstein entropy from the black hole into the Clausius entropy of the radiation field; there are no significant qualifications or limitations to this result. Throughout the evaporation process, in terms of the initial Bekenstein entropy $S_{\text{Bekenstein},0}$ we have (see Fig. 1):

$$S_{\text{Bekenstein}}(t) + S_{\text{Clausius}}(t) = S_{\text{Bekenstein},0}. \tag{5}$$

4.3. Total number of emitted Hawking quanta in GR

As a cross-check, let us estimate the total number of emitted massless quanta. We have

$$\frac{\mathrm{d}N}{\mathrm{d}M} = \frac{(\mathrm{d}N/\mathrm{d}t)}{(\hbar\langle\omega\rangle/c^2)(\mathrm{d}N/\mathrm{d}t)} = \frac{30\zeta(3)}{\pi^4}\,\frac{8\pi GM}{\hbar c}. \tag{6}$$

Integrating this we have:

$$N = \frac{30\zeta(3)}{\pi^4}\,\hat{S} \approx 0.26\,\hat{S}_2. \tag{7}$$

The total number of emitted quanta is proportional to the original Bekenstein entropy. Conversely:

$$\frac{\mathrm{d}\hat{S}_2}{\mathrm{d}N} = \frac{\pi^4}{30\zeta(3)\ln 2} \approx 3.90 \text{ bits}. \tag{8}$$

Semi-classically (at the level of macroscopic thermodynamics) everything holds together very well; the total number of massless quanta emitted over the life of the black hole is comparable to the (initial) dimensionless Bekenstein entropy.

5. Entanglement entropy: Hawking flux from a GR black hole

Now we come to the heart of the matter: Do these classical thermodynamic entropy arguments match with quantum entropy arguments based on the von Neumann entropy? If we wish to preserve unitarity, then over the lifetime of the black hole we will have to encode approximately 3.9 ± 2.5 bits per photon of hidden information into the Hawking flux. But, can we implement this “purification” process “continuously”, or is it all hidden in a (non-perturbative) burst of information at/near total evaporation? Or after the so-called Page time? [55]. We argue, assuming unitarity, complete evaporation, and a variant of the “average subsystem” argument, that the purification process is continuous and ongoing.

5.1. Entanglement: subsystem entropies

Page [56] has established a number of interesting results regarding average subsystem entropies. Consider a Hilbert space that factorizes, $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$, and on that Hilbert space consider a pure state $\rho_{AB} = |\psi\rangle\langle\psi|$. Now define subsystem density matrices via the partial traces: $\rho_i = \mathrm{tr}_j(|\psi\rangle\langle\psi|)$, where i, j runs over A, B. Then the subsystem von Neumann entanglement entropies, $\hat{S}_i = -\mathrm{tr}(\rho_i \ln \rho_i)$, both satisfy