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Bohr Model description of the critical point for the first order shape phase transition

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Abstract

The critical point of the shape phase transition between spherical and axially deformed nuclei is described by a collective Bohr Hamiltonian with a sextic potential having simultaneous spherical and deformed minima of the same depth. The particular choice of the potential as well as the scaled and decoupled nature of the total Hamiltonian leads to a model with a single free parameter connected to the height of the barrier which separates the two minima. The solutions are found through the diagonalization in a basis of Bessel functions. The basis is optimised for each value of the free parameter by means of a boundary deformation which assures the convergence of the solutions for a fixed basis dimension. Analyzing the spectral properties of the model, as a function of the barrier height, revealed instances with shape coexisting features which are considered for detailed numerical applications.

Keywords: Collective states, Shape phase transition, Critical point, Shape coexistence, Sextic potential.
PACS: 21.60.Ev, 21.10.Re, 27.70.+q

1. Introduction

The Bohr-Mottelson (BM) model [1, 2] in its extended version [3, 4, 5] represents the fundamental phenomenological frame for the description of collective behavior in atomic nuclei [6, 7]. The model's shape variables β and γ are nowadays geometrical concepts which are universally implemented where the nuclear shape is considered. The information on the actual shape of the nucleus is contained in the collective potential energy which depends in a mixed way on both β and γ. Special exactly solvable instances of the model offer few useful references in what concerns the dynamical conditions, such as the spherical vibrator [1], axially symmetric [2] and asymmetric [8] rotors. Each of these solutions, traditionally referred to as shape phases, have specific spectral characteristics emerging as a consequence of their association to the dynamical symmetries $U(5)$ [9], $SU(3)$ [10] and $O(6)$ [11] of the Interacting Boson Model (IBM) [12]. There are situations when the nucleus cannot be certainly categorized, exhibiting properties associated to different shape phases. Such nuclei are considered as critical points for a shape phase transition (SPT) undergoing along the variation of the nucleon numbers. The understanding of such transitional nuclei has considerably improved since the introduction of BM formulations with an infinite square well (ISW) potential. This is how the critical point solutions $E(5)$ [13] and $X(5)$ [14], associated with the transition from the spherical vibrator to the asymmetric and respectively symmetric rotors, came to existence. Although, analytically the two models are fairly similar, conceptually their ISW β potentials approximate quite different phenomenological pictures [15, 16]. In case of $E(5)$, the potential is just flat, marking the point of the transition between increasingly soft spherical nucleus and a deformed one. Whereas, in the $X(5)$ situation, the critical point potential has two degenerate minima, one spherical and another one deformed, separated by a small barrier. The realization of spherical or deformed shapes is then achieved by having one of the minima deeper than the other. In this context, it is said that $E(5)$ solution corresponds to the critical point for a second order SPT, while $X(5)$ for a first order.

The description of a second order SPT from one nucleus to another can be studied analytically within the Algebraic Collective Model (ACM) [17, 18, 19, 20] by employing a quartic potential, or using the quasi-exactly solvable models with a sextic potential [21, 22, 23, 24, 25]. Solving the problem for a potential corresponding to a SPT of the first order is a much harder task which cannot be achieved with neither of the mentioned approaches. The ACM diagonalization procedure based on pseudo-harmonic oscillator functions is excellent for solving Bohr Hamiltonians for a very large range of potentials which are however restrained to have a single minimum, be it flat or sharp. While, the sextic potential with two minima is quasi-exactly solvable only in special situations with limited physical relevance.

In this letter we propose a procedure to diagonalize the Bohr Hamiltonian for potentials with multiple minima. For a qualitative impact, one will focus here only on the sextic potential with two degenerated minima specific to a critical point of a first order SPT. Such a potential energy is supposed to describe two coexisting shapes. Presently,

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