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Thermal excitation spectrum from entanglement in an expanding quantum string

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ABSTRACT

A surprising result in e^+e^- collisions is that the particle spectra from the string formed between the expanding quark–antiquark pair have thermal properties even though scatterings appear not to be frequent enough to explain this. We address this problem by considering the finite observable interval of a relativistic quantum string in terms of its reduced density operator by tracing over the complement region. We show how quantum entanglement in the presence of a horizon in spacetime for the causal transfer of information leads locally to a reduced mixed-state density operator. For very early proper time τ, we show that the entanglement entropy becomes extensive and scales with the rapidity. At these early times, the reduced density operator is of thermal form, with an entanglement temperature $T_\tau = \hbar/(2\pi k_B \tau)$, even in the absence of any scatterings.

1. Introduction

A longstanding puzzle in e^+e^- collisions is that the hadron spectra measured appear thermal with features that can be characterized in terms of a common temperature [1–6]. The apparent thermal origin of the multiparticle production is surprising because scatterings appear not to be frequent enough for thermalization to occur and therefore demands an alternative explanation [2,7–10].

We argue in this letter that this apparent thermalization is an intrinsically quantum phenomenon arising from the entanglement between observable and unobservable regions in an expanding string. The observable region is described in terms of a reduced density operator by tracing over the complement region, which is bounded by the Minkowski spacetime horizon for the causal transfer of information. We show that the entanglement of the quantum vacuum accross this horizon leads to dramatic macroscopic quantum effects. In particular, for very early proper time τ, we discover that the entanglement entropy is extensive and scales with the rapidity. At these early times, a conformal symmetry emerges for the expanding system and the entanglement generates a reduced density matrix of thermal form, with the temperature

$$T_\tau = \frac{\hbar}{2\pi k_B \tau}, \tag{1}$$

even in the absence of any scatterings.

Our results establish a novel class of horizon phenomena in quantum field theory, featuring an instantaneous thermal excitation spectrum from a vacuum pure state. In contrast to the well-known example of an event horizon in the vicinity of a black hole, which leads to Hawking radiation, or the related Unruh temperature for a class of accelerated observers, our setting does not involve acceleration and it is non-stationary [11]. Specifically, the Unruh acceleration a of an observer in the Rindler-wedge of Minkowski spacetime at a spatial position $x = c^2/a$ generates a space-dependent temperature $T_x = \hbar c/(2\pi k_B x)$, while the time-dependent temperature (1) applies to the initial stages in the forward light cone with crucial applications to e^+e^- but also hadron–hadron collisions.

2. Model of expanding strings

Models that describe e^+e^- collisions successfully [12,13] rely on the Schwinger mechanism of particle production in $1+1$-dimensional quantum electrodynamics (QED); a recent comprehensive discussion of the difficulties presented by thermal-like

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spectra in such models can be found in [6]. We will work within this Schwinger model framework to treat the dynamics of the expanding string formed between the relativistic quark–antiquark pair. We choose the coordinate system such that the trajectories are in natural units $z = \pm t$, $x = y = 0$, and we assume that the strings are essentially confined to the z-direction. Bjorken coordinates are convenient, with $z = \tau \sinh(\eta)$ and $t = \tau \cosh(\eta)$ with rapidity η and proper time $\tau = \sqrt{t^2 - z^2}$. In these coordinates, the Minkowski space metric in the confined space can be expressed as $ds^2 = -d\tau^2 + \tau^2 d\eta^2$.

The Schwinger model is particularly simple for a single massless Dirac fermion. In this case, it can be bosonized to a free massive scalar theory with the action [14]

$$S = \int d^2x \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 \right\}. \tag{2}$$

For convenience, we have employed general coordinates with the two-dimensional metric $g_{\mu\nu}$. The Schwinger model bosons ϕ correspond to dipoles that are quadratic in the original fermion field. Their mass is proportional to the U(1) charge, $M = q/\sqrt{\pi}$; likewise, the string tension satisfies $\sigma = q^2/2$. Bosonization also works for a nonvanishing fermion mass m but we will not consider that case here.

3. Dynamics of expansion

For the bosonized Schwinger model, a solution corresponding to an expanding string stretched between two external quarks on their lightcones is found as a rapidity invariant solution to the equation of motion, $\partial_\tau^2 \bar{\phi} + \partial_\tau \bar{\phi}/\tau + M^2 \bar{\phi} = 0$. The boundary condition for $\tau \to 0_+$ is fixed by the requirement that the electric field $E = q\phi/\sqrt{\pi}$ approaches the U(1) charge of the external quarks $E \to q_\mathrm{e}$. This gives $\bar{\phi}(\tau) \to \sqrt{\pi} q_\mathrm{e}/q$, and with this boundary condition one finds $\bar{\phi}(\tau) = \sqrt{\pi}(q_\mathrm{e}/q) J_0(M\tau)$. The oscillations in this solution are related to multiple string breaking [15].

The solution $\bar{\phi} = \langle \phi(x) \rangle$ should be understood as a field expectation value, or equivalently, as a coherent field. Further information is contained in correlation functions of the fields $\phi(x)$ and their conjugate momentum fields $\pi(x)$, which specify a density matrix ρ at some initial time or on an appropriate Cauchy hypersurface. Because the action (2) is quadratic in the field ϕ, we also assume that this density matrix is of Gaussian form. Gaussian density matrices are fixed entirely in terms of the expectation values and connected two-point correlation functions.

4. Entanglement entropy of a rapidity interval

To discuss processes, such as the formation of hadrons or resonances during the relativistic expansion of the string, we will assume that the dynamics is local in a space-like region A within the future light cone of the spacetime instant of an e^+e^- collision. This local dynamics can be described by the reduced density matrix ρ_A, defined as the trace of ρ over the complement region B:

$$\rho_A = \mathrm{Tr}_B \rho \,. \tag{3}$$

If the fields ϕ in the regions A and B are entangled, the reduced density matrix ρ_A is of mixed form, even if the full density matrix ρ is pure. The degree of entanglement, and therefore the deviation of ρ_A from a pure state, can be characterized in terms of the entanglement entropy,

$$S_A = -\mathrm{Tr}\left\{\rho_A \ln(\rho_A)\right\} \,. \tag{4}$$

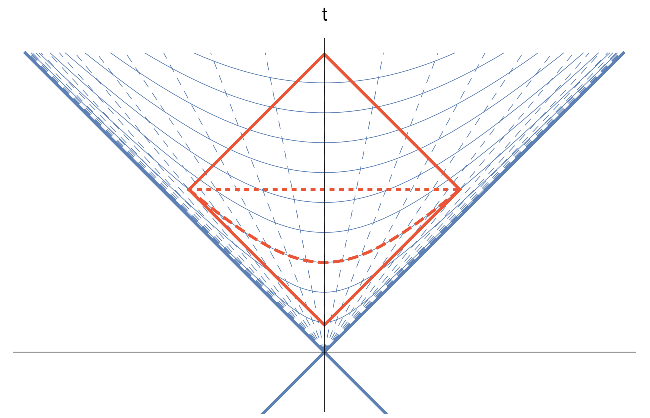

Fig. 1. Bjorken coordinates and causal development of a rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed proper time τ. The dashed red line corresponds to region A while the complement region B is composed of the rapidity intervals $(-\infty, -\Delta\eta/2)$ and $(\Delta\eta/2, \infty)$ at fixed Bjorken time τ. For $\Delta\eta \to \infty$ the causal development region approaches the lightcone. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

For most problems in quantum field theory, the determination of the reduced density matrix ρ_A, as well as of the entanglement entropy S_A, are formidable tasks. Results are currently known for free field theories in static situations [16], or conformal field theories [17–19]. For a discussion of entanglement entropy in the 't Hooft model, see [20]. The treatment of entanglement in nonequilibrium situations is especially difficult. In a companion paper [21], we develop real-time techniques for (relative) entanglement entropies of general Gaussian states in quantum field theory.

In the following, we will take advantage of the fact that if the density matrix ρ is Gaussian (in the field theoretic sense) then this is also the case for the reduced density matrix ρ_A of any equilibrium or nonequilibrium state. The entanglement entropy S_A in this case is then given by [21]

$$S_A = \frac{1}{2} \mathrm{Tr}_A \left\{ D \ln(D^2) \right\}, \tag{5}$$

where the operator trace is restricted to the region A and the matrix D consists of connected correlation functions. For the example of the bosonized Schwinger model with field ϕ and conjugate momentum field π, we obtain

$$D(x, y) = \begin{pmatrix} -i\langle \phi(x)\pi(y)\rangle_c & i\langle \phi(x)\phi(y)\rangle_c \\ -i\langle \pi(x)\pi(y)\rangle_c & i\langle \pi(x)\phi(y)\rangle_c \end{pmatrix}. \tag{6}$$

The expectation value $\langle \cdots \rangle$ in eq. (6) can equivalently be taken with respect to the full density matrix ρ or the reduced density matrix ρ_A. For the specific case of the expanding relativistic string, we will compute the trace in eq. (5) by taking A to be the rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed Bjorken time τ, corresponding to the dashed red line in Fig. 1. The complement region B corresponds to the sum of the rapidity intervals $(-\infty, -\Delta\eta/2)$ and $(\Delta\eta/2, \infty)$ at fixed Bjorken time τ. Note that any process (e.g. a measurement) within the causal development region delimited by the solid red line in Fig. 1 is by reasons of causality at most sensitive to the region A while the density matrix in the complement region B cannot affect such processes.

Because the field expectation values $\bar{\phi}(x) = \langle \phi(x) \rangle$ and $\bar{\pi}(x) = \langle \pi(x) \rangle$ do not enter (5) and (6), the entanglement entropy for an expanding string described by the massless Schwinger model (corresponding to a coherent state specified by $\bar{\phi}(x)$ and $\bar{\pi}(x)$) is the same as the one of the vacuum (which is a coherent state with vanishing field expectation values). Moreover, the entropy does not change under unitary evolution with the boundary