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Physics Letters B
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Spatial distribution of angular momentum inside the nucleon

Cédric Lorcé [a], Luca Mantovani [b,c], Barbara Pasquini [b,c,*]

[a] Centre de Physique Théorique, École polytechnique, CNRS, Université Paris-Saclay, F-91128 Palaiseau, France
[b] Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy
[c] Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy

ARTICLE INFO

Article history:
Received 7 May 2017
Received in revised form 18 October 2017
Accepted 9 November 2017
Available online xxxx
Editor: J.-P. Blaizot

Keywords:
Energy–momentum tensor
Angular momentum
Spin structure of the nucleon

ABSTRACT

We discuss in detail the spatial distribution of angular momentum inside the nucleon. We show that the discrepancies between different definitions originate from terms that integrate to zero. Even though these terms can safely be dropped at the integrated level, they have to be taken into account when discussing distributions. Using the scalar diquark model, we illustrate our results and, for the first time, check explicitly that the equivalence between kinetic and canonical orbital angular momentum persists at the level of distributions, as expected in a system without gauge degrees of freedom.

1. Introduction

Understanding how the spin of the nucleon originates from the spin and orbital motion of its constituents is one of the current key questions in hadronic physics. While this problem may seem rather straightforward in the context of ordinary quantum mechanics, it becomes quite challenging in the context of hadronic physics, where one has to include relativistic, gauge-symmetry and non-perturbative aspects. One of the main conceptual issues is that the decomposition of the nucleon spin is not unique [1–3]. This intrinsic ambiguity is sometimes considered as a sign indicating that the question is not physical. It actually reflects the fact that any decomposition necessarily relies on how one defines the degrees of freedom. The problem remains physical as long as the various contributions can in principle be accessed by experiments.

Ji has shown that the (kinetic) total angular momentum of quarks and gluons can be expressed in terms of generalized parton distributions (GPDs) [4]. This triggered an intense experimental program since GPDs can be extracted from exclusive processes like deeply virtual Compton scattering and hard meson exclusive electroproduction [5–7]. Interestingly, the connection between GPDs and angular momentum has been clearly established only at the level of integrated quantities over all space. As shown by Burkardt [8,9], GPDs contain information about the spatial distribution of quarks and gluons inside the nucleon. It is therefore conceivable that GPDs contain also the information about the spatial distribution of angular momentum. The problem now is to determine how this information is precisely encoded.

Polyakov provided the first attempt to answer this question [10], but he required the nucleon to be infinitely massive, so as to avoid relativistic corrections. The infinite mass assumption can actually be relaxed, provided that one works within the light-front formalism, as sketched in the review [1]. Recently, Adhikari and Burkardt compared different definitions of the angular momentum distribution and reached the conclusion that none of them agree. They attributed some of the discrepancies to missing total divergence terms, as it had been pointed out earlier in Refs. [1,3].

The purpose of the present paper is to revisit the work of Polyakov, discuss in more detail the alternative approach based on the light-front formalism, and identify all the missing terms that hinder the proper comparison of the various definitions of angular momentum.

The rest of the paper is organized as follows. In Section 2, we recall the connection between the energy–momentum tensor and angular momentum. We stress in particular that, unlike in General Relativity, the energy–momentum tensor is generally not symmetric in Particle Physics, owing to the presence of a spin density. In Section 3, we derive three-dimensional distributions of angular momentum in the Breit frame. We show that by projecting these distributions onto a two-dimensional plane, they can be considered in the more general class of elastic frames. In Section 4, we discuss

* Corresponding author at: Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy.
E-mail address: barbara.pasquini@pv.infn.it (B. Pasquini).

https://doi.org/10.1016/j.physletb.2017.11.018

the distributions in the light-front formalism and observe that they coincide (for the longitudinal component of angular momentum) with the two-dimensional distributions in the elastic frame. We illustrate our results within the scalar diquark model in Section 5 and, for the first time, check explicitly that kinetic and canonical orbital angular momentum coincide at the level of distributions in absence of gauge bosons. Finally, in Section 6 we summarize our findings and draw our conclusions.

2. Energy–momentum and generalized angular momentum tensors

In field theory, the conserved current associated with the invariance of the theory under Lorentz transformations, known as generalized angular momentum tensor, can be written in general as the sum of two contributions

$$J^{\mu\alpha\beta}(x) = L^{\mu\alpha\beta}(x) + S^{\mu\alpha\beta}(x)\,. \tag{1}$$

Each one of these tensors is antisymmetric under $\alpha \leftrightarrow \beta$. The first contribution reads

$$L^{\mu\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x)\,, \tag{2}$$

where $T^{\mu\nu}(x)$ is the Energy–Momentum Tensor (EMT) density associated with the system and accounts for the fact that the fields are affected by Lorentz transformations owing to their dependence on space–time points. The second contribution $S^{\mu\alpha\beta}(x)$ accounts for the fact that fields have in general many components, which can also be affected by Lorentz transformations.

The three generators of rotations are obtained when $\alpha, \beta = i, j$ are spatial components. In this case, Eq. (1) simply indicates that the total Angular Momentum (AM) is the sum of Orbital Angular Momentum (OAM) and spin

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}\,, \tag{3}$$

with $J^i = \frac{1}{2}\epsilon^{ijk}\int d^3\boldsymbol{r}\, J^{0jk}$, likewise for L^i and S^i.

2.1. Belinfante-improved tensors

The energy–momentum tensor obtained by following the procedure in Noether's theorem is referred to as the canonical EMT, and is in general neither gauge invariant nor symmetric. Belinfante and Rosenfeld [11–13] proposed to add a so-called superpotential term to the definition of both the energy–momentum and generalized angular momentum tensors, defining the Belinfante-improved tensors as

$$T^{\mu\nu}_{\text{Bel}}(x) = T^{\mu\nu}(x) + \partial_\lambda G^{\lambda\mu\nu}(x), \tag{4}$$

$$J^{\mu\alpha\beta}_{\text{Bel}}(x) = J^{\mu\alpha\beta}(x) + \partial_\lambda\left[x^\alpha G^{\lambda\mu\beta}(x) - x^\beta G^{\lambda\mu\alpha}(x)\right]\,, \tag{5}$$

where the superpotential $G^{\lambda\mu\nu}$ is given by the combination

$$G^{\lambda\mu\nu}(x) = \frac{1}{2}\left[S^{\lambda\mu\nu}(x) + S^{\mu\nu\lambda}(x) + S^{\nu\mu\lambda}(x)\right] = -G^{\mu\lambda\nu}(x)\,. \tag{6}$$

The effect of such a term is to modify the definition of the local density without changing the total charge. The Belinfante-improved tensors (4)–(5) are conserved and usually turn out to be gauge invariant. Moreover, the particular choice (6) allows us to write the total AM in a pure orbital form

$$J^{\mu\alpha\beta}_{\text{Bel}}(x) = x^\alpha T^{\mu\beta}_{\text{Bel}}(x) - x^\beta T^{\mu\alpha}_{\text{Bel}}(x). \tag{7}$$

Since the new tensors are conserved, it follows from this expression that the Belinfante-improved EMT is symmetric.

2.2. Kinetic tensors

As discussed in Refs. [1,14], the requirement of a symmetric EMT is usually motivated by General Relativity. In that context, the notion of spin is not accounted for from the beginning, and it is natural to consider AM as purely orbital. From a Particle Physics perspective, however, one naturally includes a spin contribution to the total AM as in Eq. (1). It then follows from the conservation of both $T^{\mu\nu}(x)$ and $J^{\mu\alpha\beta}(x)$ that the EMT is in general asymmetric, the antisymmetric part being given by the divergence of the density of spin

$$T^{[\alpha\beta]}(x) = -\partial_\mu S^{\mu\alpha\beta}(x), \tag{8}$$

where $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$. We see the Belinfante-improved tensors as effective densities, where the effects of spin are mimicked by an obscure new contribution to momentum. Interestingly, recent developments in optics also seem to demote the Belinfante-improved expressions from their status as fundamental densities [15].

Instead of the Belinfante-improved tensors, Ji [4] proposed to use in the context of QCD the kinetic EMT

$$T^{\mu\nu}_{\text{kin}}(x) = T^{\mu\nu}_{\text{kin},q}(x) + T^{\mu\nu}_{\text{kin},g}(x), \tag{9}$$

where the gauge-invariant quark and gluon contributions are given by [1,14]

$$T^{\mu\nu}_{\text{kin},q}(x) = \frac{1}{2}\,\overline{\psi}(x)\gamma^\mu i\overleftrightarrow{D}^\nu\psi(x)\,, \tag{10}$$

$$T^{\mu\nu}_{\text{kin},g}(x) = -2\,\text{Tr}\left[G^{\mu\lambda}(x)G^{\nu}{}_{\lambda}(x)\right] + \frac{1}{2}\,g^{\mu\nu}\,\text{Tr}[G^{\rho\sigma}(x)G_{\rho\sigma}(x)]\,, \tag{11}$$

with $\overleftrightarrow{D}^\mu = \overleftrightarrow{\partial}^\mu - 2igA^\mu$ and $\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$, and the field-strength tensor $G_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig\left[A_\mu(x), A_\nu(x)\right]$. The kinetic generalized AM tensor reads

$$J^{\mu\alpha\beta}_{\text{kin}}(x) = L^{\mu\alpha\beta}_{\text{kin},q}(x) + S^{\mu\alpha\beta}_q(x) + J^{\mu\alpha\beta}_{\text{kin},g}(x)\,, \tag{12}$$

with

$$L^{\mu\alpha\beta}_{\text{kin},q}(x) = x^\alpha T^{\mu\beta}_{\text{kin},q}(x) - x^\beta T^{\mu\alpha}_{\text{kin},q}(x)\,, \tag{13}$$

$$S^{\mu\alpha\beta}_q(x) = \frac{1}{2}\,\varepsilon^{\mu\alpha\beta\lambda}\,\overline{\psi}(x)\gamma_\lambda\gamma_5\psi(x)\,, \tag{14}$$

$$J^{\mu\alpha\beta}_{\text{kin},g}(x) = x^\alpha T^{\mu\beta}_{\text{kin},g}(x) - x^\beta T^{\mu\alpha}_{\text{kin},g}(x)\,, \tag{15}$$

and the convention $\varepsilon_{0123} = +1$. Contrary to the quark total AM, the gluon total AM cannot be split into orbital and spin contributions, which are at the same time gauge-invariant and local [16,17]. The kinetic and Belinfante-improved tensors in QCD are related as follows

$$T^{\mu\nu}_{\text{kin},q}(x) = T^{\mu\nu}_{\text{Bel},q}(x) - \frac{1}{2}\,\partial_\lambda S^{\lambda\mu\nu}_q(x)\,, \tag{16}$$

$$L^{\mu\alpha\beta}_{\text{kin},q}(x) + S^{\mu\alpha\beta}_q(x) = J^{\mu\alpha\beta}_{\text{Bel},q}(x) - \frac{1}{2}\,\partial_\lambda\left[x^\alpha S^{\lambda\mu\beta}_q(x) - x^\beta S^{\lambda\mu\alpha}_q(x)\right]\,, \tag{17}$$

the gluon contributions being the same in both cases, $T^{\mu\nu}_{\text{kin},g}(x) = T^{\mu\nu}_{\text{Bel},g}(x)$ and $J^{\mu\alpha\beta}_{\text{kin},g}(x) = J^{\mu\alpha\beta}_{\text{Bel},g}(x)$. Using the conservation of the total AM $J^{\mu\alpha\beta}_{\text{kin}}$ and the symmetry of $T^{\mu\nu}_{\text{kin},g}(x)$, one can relate the antisymmetric part of the quark kinetic EMT to the quark spin divergence

$$T^{[\alpha\beta]}_{\text{kin},q}(x) = -\partial_\mu S^{\mu\alpha\beta}_q(x), \tag{18}$$