Physics Letters B 776 (2018) 58–63

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Diffusion constant of slowly rotating black three-brane

Z. Amoozad, J. Sadeghi *

*Sciences Faculty, Department of Physics, University of Mazandaran, P.O. Box 47416-95447, Babolsar, Iran*

A R T I C L E   I N F O

*Article history:*
Received 11 July 2017
Received in revised form 11 October 2017
Accepted 13 November 2017
Available online 20 November 2017
Editor: N. Lambert

*Keywords:*
Rotating black three-brane
Diffusion constant
Fick's laws

A B S T R A C T

In this paper, we take the slowly rotating black three-brane background and perturb it by introducing a vector gauge field. We find the components of the gauge field through Maxwell equations and Bianchi identities. Using currents and some ansatz we find Fick's first law at long wavelength regime. An interesting result for this non-trivial supergravity background is that the diffusion constant on the stretched horizon which emerges from Fick's first law is a complex constant. The pure imaginary part of the diffusion constant appears because the black three-brane has angular momentum. By taking the static limit of the corresponding black brane the well known diffusion constant will be recovered. On the other hand, from the point of view of the Fick's second law, we have the dispersion relation $\omega = -iDq^2$ and we found a damping of hydrodynamical flow in the holographically dual theory. Existence of imaginary term in the diffusion constant introduces an oscillating propagation of the gauge field in the dual field theory.



## 1. Introduction

In ten dimensional type IIB supergravity a black three-brane is a cluster of coincident $D3$-branes [1]. Black three-branes may have angular momentum in a plane perpendicular to the brane and the geometry of this object has extensively been discussed in [2–4]. Generally a three-brane in ten dimensions has a rank three transverse rotation group $SO(6)$, so it has three independent (commuting) angular momentums.

An interesting aspect of $D3$-brane is that worldvolume of $N$ coincident $D3$-branes, being in ten dimensional spacetime and at low energies, can be described by super-Yang–Mills theory with gauge group $U(N)$. This point shows that $D3$-branes and black three-branes might be used in the context of dual theories. The dual theory which connects physics in a region of space with gravity to a region without gravity; is known as holographic principle.

In the context of the holographic principle [5–7], the region without gravity which lives on smaller dimensions, can describe in particular cases some kinds of field theories. As we know, the fluid/gravity duality connects the long wavelength field theories to a gravity dual containing a black brane with nonzero Hawking temperature [8,9]. It has been proved that theories living on the non extremal $D3$, $M2$ and $M5$ branes have hydrodynamical behavior at long wavelength description and their hydrodynamic modes are coincide with kinetic coefficients in the dual theories extracted from AdS/CFT [10–14].

On the other hand, the membrane paradigm, which describes the hydrodynamic-like properties of the event horizon, would be a better physical model for black branes in comparison with black holes because the horizon of black holes lacks translational invariants [15,16]. Also considering fluctuations around static black brane solutions, the diffusion relations and shear flow in the holographically dual theory have been extracted in [17,18].

Some properties of the rotating black three-brane such as thermodynamic properties and stability has been studied in the literature [19–21]. Also the relation between rotating black three-branes and $QCD$ has been calculated in [22]. The points mentioned above motivated us to investigate dispersion relation of dual field theory of rotating black three-brane spacetime which the hydrodynamical modes are encoded in. It helps us to understand more about fluid/gravity duality for the case of rotating branes. So we take slowly rotating black three-brane in the gravity side and introduce a small fluctuation, using a vector gauge field from the field theory side, and find Fick's first law. By extracting Fick's first law, an explicit expression for the diffusion constant will be obtained. Using the Fick's second law, i.e. $\omega = -iDq^2$, we find a traditional damping of the hydrodynamical modes (because of the real part of the diffusion constant) plus an oscillating propagation of the gauge field, which emerges from the imaginary part of the obtained dif-

* Corresponding author.
*E-mail addresses:* z.amoozad@stu.umz.ac.ir (Z. Amoozad), pouriya@ipm.ir (J. Sadeghi).

https://doi.org/10.1016/j.physletb.2017.11.024

fusion constant, in long wave length regime of the corresponding field theory.

The organization of the paper is as follows. In section 2, we introduce the reduced spacetime of rotating black three-brane. By using the gauge field, we perturb this background and then apply Maxwell equations and Bianchi identities on the stretched horizon. In that case, in order to determine all appropriate components of the gauge field we suppose some ansatz. In section 3, we will obtain Fick's first law by relating different components of Maxwell equations. Then, we extract an explicit expression for the diffusion constant of the dual field theory. Section 4 is devoted to the Fick's second law and the dispersion relation which relates the quasi normal modes of black branes to the diffusion constant. In the final section we have conclusions and notes.

## 2. Rotating black three-brane and corresponding perturbation

The general rotating brane solution in string theory is presented in [21,22]. So thermodynamic properties and stability conditions of that solutions can be found easily. One of the hydrodynamic modes is the diffusion constant and in order to find it we consider the reduced metric of the rotating three-brane with angular momentum (here we have just one angular momentum),

$$ds^2_{IIB} = f^{-\frac{1}{2}}\left\{-hdt^2 + dx_1^2 + dx_2^2 + dx_3^2\right\} + f^{\frac{1}{2}}\left\{\frac{dr^2}{\widetilde{h}} + r^2(\Delta d\theta^2 + \widetilde{\Delta} sin^2\theta d\Phi^2 + cos^2\theta d\Omega_3^2) - \frac{2lr_0^4 cosh\alpha}{r^4\Delta f} sin^2\theta dt d\Phi\right\}, \tag{1}$$

in which,

$$f = 1 + \frac{r_0^4 sinh^2\alpha}{r^4\Delta},$$

$$\Delta = 1 + \frac{l^2 cos^2\theta}{r^2}, \qquad \widetilde{\Delta} = 1 + \frac{l^2}{r^2} + \frac{r_0^4 l^2 sin^2\theta}{r^6\Delta f},$$

$$h = 1 - \frac{r_0^4}{r^4\Delta}, \qquad \widetilde{h} = \frac{1}{\Delta}\left(1 + \frac{l^2}{r^2} - \frac{r_0^4}{r^4}\right), \tag{2}$$

and we have assumed [22],

$$d\Omega_3^2 = d\Psi_1^2 + sin^2\Psi_1 d\Psi_2^2 + cos^2\Psi_1 d\Psi_3^2. \tag{3}$$

Similar to spherically symmetric cases the dilaton is constant and $cosh\alpha$ relates directly to the three-brane charge. As we know, the rotating black three-brane and $N$ coincident $D3$-branes with a density of $R$-charge on the world-volume have the same quantum numbers. Thus we have,

$$r_0^4 sinh\alpha cosh\alpha = R^4 \equiv 4\pi g_s N\alpha'^2, \tag{4}$$

where $g_s$ and $\alpha'$ are the string theory parameters. The event horizon for the metric (1) is,

$$r_e^2 = \frac{1}{2}\left\{-l^2 + \sqrt{l^4 + 4r_0^4}\right\}, \tag{5}$$

where the Hawking temperature at $\theta = \frac{\pi}{2}, \Psi_1 = 0$ is given by,

$$T\mid_{r=r_e} = \frac{r_e}{2\pi r_0^4 cosh\alpha}(2r_e^2 + l^2). \tag{6}$$

In the slowly rotating limit (small angular momentum) we take $\frac{l^2}{r_0^2} \ll 1$ and higher order terms can be neglected. So $r_e^2 = r_0^2(1 - \frac{l^2}{2r_0^2}) + O(\frac{L^4}{r_0^4})$, and the temperature is $T\mid_{r=r_e} = \frac{1}{\pi r_0 cosh\alpha}(1 - \frac{l^2}{4r_0^2})$.

It is obvious that (4) connects $r_0$ and $cosh\alpha$ to the $D3$-brane charge $N$. So, in the low energy limit, i.e. $N \to \infty$, we have $cosh\alpha = (\frac{4\pi g_s\alpha'^2}{r_0^4}N)^{\frac{1}{2}} = \frac{R^2}{r_0^2}$ and the Howking temperature will be,

$$T_H|_{N\to\infty} = \frac{r_0}{\pi(4\pi g_s\alpha'^2 N)^{\frac{1}{2}}}(1 - \frac{l^2}{4r_0^2}) = \frac{r_0}{\pi R^2}(1 - \frac{l^2}{4r_0^2}). \tag{7}$$

This relation expresses that in the large 't Hooft coupling ($\lambda = g_{YM}^2 N \gg 1$), Howking temperature of rotating three-brane in the static limit gives the $AdS5$ result exactly.

The stretched horizon is a suitable region to calculate the diffusion constant on. In our case, the stretched horizon is a flat spacelike hypersurface located at $r = r_h$ such that,

$$r_h > r_e, \qquad r_h - r_e \ll r_e. \tag{8}$$

We find in the next sections a more restricted constraint than (8). The unit normal vector on the corresponding hypersurface is a spacelike vector. Using $\Phi = r = const.$ we have $n_r = \sqrt{g_{rr}}$. Also the inverse components of the metric $ds^2_{IIB}$ are,

$$g^{tt} = \frac{-g_{\Phi\Phi}}{g_{t\phi}^2 - g_{tt}g_{\Phi\Phi}}, \qquad g^{\Phi\Phi} = \frac{-g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\Phi\Phi}},$$

$$g^{t\Phi} = \frac{g_{t\Phi}}{g_{t\phi}^2 - g_{tt}g_{\Phi\Phi}},$$

$$g^{x_ix_i} = (g_{x_ix_i})^{-1}, \qquad g^{rr} = (g_{rr})^{-1},$$

$$g^{\theta\theta} = (g_{\theta\theta})^{-1}, \qquad g^{\Psi_i\Psi_i} = (g_{\Psi_i\Psi_i})^{-1}. \tag{9}$$

The hydrodynamic theory in the dual field theory can be produced by currents and tensors, so it is important to investigate the vector and tensor perturbations. The dynamics of vector perturbation can be found by Maxwell action,

$$S_{gauge} \sim \int dx^{p+2}\sqrt{-g}(\frac{1}{g_{eff}^2}F^{\mu\nu}F_{\mu\nu}). \tag{10}$$

Taking an external gauge field $A_\mu$ as a perturbation, one can study currents on the stretched horizon $r = r_h$. The corresponding equation of current is,

$$J^\mu = n_\nu F^{\mu\nu}\mid_{r_h}, \tag{11}$$

after expansion we have,

$$J^t = \frac{g^{tt}}{\sqrt{g_{rr}}}F_{tr}, \qquad J^{x_i} = \frac{1}{g_{x_ix_i}\sqrt{g_{rr}}}F_{x_ir}, \qquad J^\theta = \frac{1}{g_{\theta\theta}\sqrt{g_{rr}}}F_{\theta r},$$

$$J^\Phi = \frac{g^{\Phi\Phi}}{\sqrt{g_{rr}}}F_{\Phi r}, \qquad J^{\Psi_i} = \frac{1}{g_{\Psi_i\Psi_i}\sqrt{g_{rr}}}F_{\Psi_i r}. \tag{12}$$

The conservation equation of current is $\partial_\mu J^\mu = 0$, this leads us to the following equation,

$$\frac{g^{tt}}{\sqrt{g_{rr}}}\partial_t F_{tr} + \frac{1}{g_{x_ix_i}\sqrt{g_{rr}}}\partial_{x_i}F_{x_ir} + \partial_\theta(\frac{F_{\theta r}}{g_{\theta\theta}\sqrt{g_{rr}}}) = 0. \tag{13}$$

Because of the antisymmetry properties of $F^{\mu\nu}$ one can conclude $n_\mu J^\mu = 0$, which shows that the corresponding current is parallel to the stretched horizon.

Now we search for the other relations come from Maxwell equations and Bianchi identities. For the Maxwell equation we have,

$$\partial_\mu(\frac{1}{g_{eff}^2}\sqrt{-g}F^{\mu\nu}) = 0. \tag{14}$$

For simplicity, we take effective coupling $g_{eff}$ as a constant, so the components of the Maxwell equations will be,