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Casimir scaling and Yang–Mills glueballs

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ABSTRACT

We conjecture that in Yang–Mills theories the ratio between the ground-state glueball mass squared and the string tension is proportional to the ratio of the eigenvalues of quadratic Casimir operators in the adjoint and the fundamental representations. The proportionality constant depends on the dimension of the space-time only, and is henceforth universal. We argue that this universality, which is supported by available lattice results, is a direct consequence of area-law confinement. In order to explain this universal behavior, we provide three analytical arguments, based respectively on a Bethe–Salpeter analysis, on the saturation of the scale anomaly by the lightest scalar glueball and on QCD sum rules, commenting on the underlying assumptions that they entail and on their physical implications.

1. Introduction

Yang–Mills (YM) theories without matter fields are believed to exhibit a confining phase at low energies, in which all bound states (glueballs) are gapped and color-singlet. Confinement in YM theories is supported by lattice studies [1]. However, since glueballs are nonperturbative objects, we do not have yet good understanding of the properties of glueballs such as their mass spectrum or decay widths.

It has been suggested that color confinement can be described in terms of a dual Higgs mechanism or monopole condensation [2–4]. In this picture, monopoles, dual to color charges, condense in the color-confined phase, and 't Hooft operators develop a vacuum expectation value. The dynamical scale κ is set by the condensate, which should be responsible for all other dimensional quantities in the confined phase. Monopole condensation implies a linear potential between a pair of static color charges, or equivalently an area law for the Wilson loop.

In this letter we provide theoretical arguments and numerical evidence for the existence of a new universal law. The law states that the ratio of ground state glueball mass squared and the string tension is universally proportional to the ratio of the eigenvalues of quadratic Casimir operators for all confining gauge theories. The proportionality constant is independent of the gauge group and the strength of coupling as long as the area law arises. It depends only on the dimensionality of the space-time.

2. Glueball mass

Calculating the ground state glueball mass is tantamount to showing that there is a gap in the ground state of pure YM theory, which has never been proved analytically except in three dimensions [5]. Numerical calculations of glueball masses on the lattice show the existence of a gap in YM theories [6–13].

Asymptotically, for confining YM theories, the expectation value of rectangular Wilson loops $\mathcal{C}$ can be written as

$$\langle W(\mathcal{C})\rangle = \left\langle \frac{1}{N} e^{i\oint_{\mathcal{C}} A} \right\rangle = \exp\left[-\sigma LT + \cdots\right], \tag{1}$$

where LT is the area of $\mathcal{C}$, σ is the string tension between a static quark–antiquark pair, and the ellipsis includes subleading corrections such as the Lüscher term. Following the area law confinement, we write the string tension σ as to define κ via the proportionality to the quadratic Casimir operator on the fundamental representation

$$\sigma = \kappa^2 C_2(F)\,, \tag{2}$$

which is consistent with lattice results [8,14–18]. The glueball is a bound state of adjoint gluons. On dimensional grounds, its mass should be proportional to κ. For the ground state glueball we conjecture

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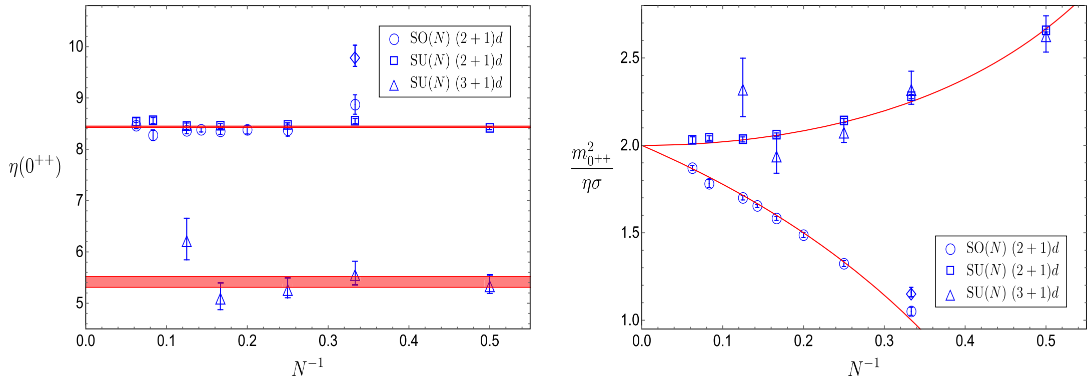

Fig. 1. The universal ratio η (left panel), and glueball masses squared in units of the string tension (right panel), for various YM theories as a function of $1/N$. The solid curves are the Casimir ratio $C_2(A)/C_2(F)$ for SU(N) (upper curve) and SO(N) (lower curve), respectively. The value of η from the tension of the SO(3) fundamental string is marked as $\diamond$.

$$m^2_{0^{++}} = \eta\,\kappa^2\,C_2(A)\,, \tag{3}$$

where η is a universal ratio and $C_2(A)$ the quadratic Casimir for the adjoint representation. The existence of the universal ratio η is consistent with the large-N universality of YM theories, supported by Wilson loop calculations [19] and gauge-gravity dualities [20]. At finite N, the ratio of the eigenvalues of the relevant quadratic Casimir operators is [21]

$$\frac{C_2(A)}{C_2(F)} = \begin{cases} \frac{2N^2}{N^2-1} & \text{for SU}(N) \\ \frac{2(N-2)}{N-1} & \text{for SO}(N) \\ \frac{4(N+1)}{2N+1} & \text{for Sp}(2N)\,, \end{cases} \tag{4}$$

and approaches 2 in the large-N limit.

Glueball masses and string tensions have been calculated by various collaborations for YM theories in $3+1$ and $2+1$ dimensions [6–13]. From the continuum-extrapolated lattice results of glueball mass and string tension, taking the data from the most recent large-N calculations available in the literature [8,11,13] (Fig. 1), we find[1]

$$\eta(0^{++}) \equiv \frac{m^2_{0^{++}}}{\sigma}\cdot\frac{C_2(F)}{C_2(A)} = \begin{cases} 5.41(12), & (d=3+1)\,, \\ 8.440(14)(76), & (d=2+1)\,. \end{cases} \tag{5}$$

For $3+1$ dimensions Eq. (5) is the constant fit of SU(N) results over $2 \le N \le 8$, with χ^2/d.o.f. $\simeq 1$. For $2+1$ dimensions, lattice results are available for SU(N), as well for SO(N), with $2 \le N \le 16$, hence we performed a constant fit for the universal ratio η of both data sets.[2] The resulting statistical error is quoted in the first parentheses in Eq. (5), with somewhat larger value of χ^2/d.o.f. $\simeq$ 1.9.[3]

Deviations from universality in $2+1$ dimensions between two classes of gauge groups are assessed by calculating η separately. We find $\eta = 8.386(25)$ (χ^2/d.o.f. $\simeq 1.3$) for SO(N) and $\eta = 8.462(16)$ (χ^2/d.o.f. $\simeq 1.9$) for SU(N). Given the expectation that the large-N limit of the two sets should coincide, this difference of 3σ level is probably due to the systematic errors in the lattice data. We account for the discrepancy with a systematic error reported in the second parenthesis in Eq. (5). We also studied two heavier states, the 2^{++} glueball and the first excited scalar glueball, 0^{*++}. The excited states start to see the deviation from the area-law confinement, hence it is not surprising that the 0^{*++} does not show universal behavior. (See Fig. 2.) For the 2^{++}, however, it is inclusive, because the constant fit gives a poor χ^2/d.o.f. $\simeq 19$ for the 2^{++} tensor glueballs in $2+1$ dimensions, while it fits much better in $3+1$ dimensions with χ^2/d.o.f. $\simeq 1.1$.

3. Glueball mass and Casimir scaling

Motivated by the strong numerical evidence for Casimir scaling, we provide three analytical arguments to explain its origin. None of the arguments is fully conclusive, as they all rely on specific dynamical assumptions that we highlight explicitly, yet the picture that emerges is that Casimir scaling of ground state mass should capture much of the essence of the confinement properties of YM theories.

3.1. Bethe–Salpeter equation

The amplitude for creating two gluons out of vacuum to form a color-singlet bound state of momentum P with a polarization λ can be defined as

$$\Gamma^{\mu\nu}_R(x_1,x_2;P,\lambda) = \langle 0|\,\mathrm{T}A^{\mu\,a}(x_1)A^{\nu\,a}(x_2)\,|R(P,\lambda)\rangle\,, \tag{6}$$

where T denotes the time-ordered product and $\langle 0|$ is the vacuum. Summation over color indices a is understood.

The bound state amplitude satisfies the Bethe–Salpeter (BS) equations, obtained from the gluon four-point scattering amplitude near the pole, which are diagrammatically shown for the amputated BS amplitude in Fig. 3.

From the BS equation, the scalar (amputated) amplitude χ_P obeys, in Euclidean space,

$$\left[\partial^2 - P^2\right]\chi_P(x) = \int \mathrm{d}^4y\,V(x-y)\chi_P(y)\,, \tag{7}$$

with $x = x_1 - x_2$ the displacement of two external gluons.

The area law for confinement is associated with the Regge behavior of the spectrum: $M_n^2 \sim n$, where $n = 1, 2, \cdots$ are the radial quantum numbers, reproduced by the approximate BS kernel

$$V(x-y) \approx \frac{1}{2}\omega^2 x^2\,\delta^4(x-y)\,. \tag{8}$$

[1] Our conjecture for the universal ratio is also supported by the analytic calculation of the ground-state glueball mass in $2+1$ dimensional SU(N) gauge theories [22], which finds $\eta(0^{++}) \simeq 8.41$, and suspected in the constituent gluon model in [23].

[2] The string tension can be defined also for SO(3) by considering distances of the order of the confinement scale. Yet, it is affected by large systematic uncertainties due to its instability [11,13]. To mitigate the systematics, instead of this quantity, we use the string tension obtained from the fundamental of SU(2), assuming Casimir scaling for the string tension. We checked that by using the measured value of the string tension of SO(3), the value of η does not change but yields a poor χ^2/d.o.f $\simeq$ 4.8.

[3] The χ^2 distribution does not improve significantly, even if the data for the lowest N is excluded.