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Quantum corrections to thermodynamics of quasitopological black holes

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ABSTRACT

Based on the modification to area-law due to thermal fluctuation at small horizon radius, we investigate the thermodynamics of charged quasitopological and charged rotating quasitopological black holes. In particular, we derive the leading-order corrections to the Gibbs free energy, charge and total mass densities. In order to analyze the behavior of the thermal fluctuations on the thermodynamics of small black holes, we draw a comparative analysis between the first-order corrected and original thermodynamical quantities. We also examine the stability and bound points of such black holes under effect of leading-order corrections.

1. Overview and motivation

According to AdS/CFT duality, the Einstein general relativity in the bulk space–time corresponds to a gauge theory living on the boundary with a large N (number of colors) and large 't Hooft coupling [1]. Since the coupling constants in the gravity side relate to central charges in the gauge theory, therefore Einstein gravity has limited number of dual CFTs, in particular only those CFTs which have equal central charges, as Einstein gravity does not have enough free parameters. The presence of various higher-order derivatives in AdS gravity corresponds to new couplings among operators in the dual CFT. One well-known example of higher derivatives gravity theories is Gauss–Bonnet gravity. The Gauss–Bonnet gravity involves only one quadratic coupling term and therefore the corresponding range of dual theory is still limited. In order to improve this limitation of holographic studies to the classes of CFTs, one has to introduce the new higher order curvature terms, at least curvature-cubed terms, into gravity. One may achieve such curvature-cubed interactions by adding the cubic term in Lovelock gravity [2], but can not be very helpful as such term is topological in nature and becomes significant only in very high dimensions.

Recently, a new toy model for gravitation action has been proposed which contains not only the Gauss–Bonnet term but also a curvature-cubed interaction [3,4]. This is a quasitopological gravity model as the cubic terms do not have a topological origin like Lovelock gravity but contribute dynamically to the evolution of fields in the bulk. This quasitopological gravity theory is endowed with two important properties. First, the equations of motion are generically of fourth order in derivatives of the metric which reduces to second order for spherically symmetric space-times, and second there exist the exact black hole solutions [4]. The holographic discussions for these black hole solutions with some recipes of AdS/CFT duality have been given in [5]. The basic thermodynamics of quasitopological Reissner-Nordström black holes are studied in Ref. [6]. Recently, the surface term of quasitopological gravity for space–time with flat boundary is introduced and the thermodynamic properties of these solutions have been investigated by using the relation between on-shell action and Gibbs free energy [7].

An important discovery that black holes behave as thermodynamic objects had affected our understanding of gravity theory and its relationship to quantum field theory considerably. Bekenstein and Hawking were first who proposed that black holes radiate as black bodies with characteristic entropy related to the area of the horizon [8]. In present scenario, it is more or less certain that black holes much larger than the Planck scale have entropy proportional to its horizon area [8–12]. So, this poses an interesting question that what could be the leading-order corrections when one reduces the size of the black holes. To answer this question, several attempts have been made. For instance, using a corrected version of the asymptotic Cardy formula for BTZ, string theoretic and all other black holes, whose microscopic degrees of freedom are described by an underlying CFT [13], the leading-order corrections have found logarithmic in nature. In fact, the consideration of matter fields in black hole backgrounds also yields logarithmic

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correction to the black holes entropy at the leading order [14]. The leading-order correction to black holes entropy is also logarithmic by considering string-black hole correspondence [15,16] and using Rademacher expansion of the partition function [17]. Furthermore, Das et al. in Ref. [19] showed that the leading-order corrections to the entropy of any thermodynamic system due to small statistical fluctuations around equilibrium are always logarithmic.

The study of leading-order correction to the black holes thermodynamics is a subject of current interests. In this direction, recently, the effects of quantum corrections on thermodynamics and stability of Gödel black hole [20], Schwarzschild–Beltrami–de Sitter black hole [21] and massive black hole in AdS space [22] have been studied. The corrected thermodynamics of a dilatonic black hole has also been discussed [23] which meets the same universal form of correction term. In another work, the corrected thermodynamics of a black hole is also studied from the partition function points of view [24]. The quantum gravity effects on the Hořava–Lifshitz black hole thermodynamics are analyzed and their stability is also discussed [25]. Similar investigation in case of the modified Hayward black hole is also made, where it has been found that correction term reduces the pressure and internal energy of the Hayward black hole [26]. We try to extend such study to the case of quasitopological black holes.

In this paper, we consider a charged quasitopological model which exhibits black hole solutions and discuss the effects of leading-order correction on thermodynamics which becomes significant for small size of the black holes. First, we compute the leading-order correction to the entropy of charged quasitopological black hole and plot a graph to make a comparative analysis between corrected and uncorrected entropy densities for smaller black holes. Here, we find that for (negative-)positive correction parameter (α) there exists a (positive-)negative peak for the corrected entropy density at sufficiently small black holes. The corrected entropy density becomes negative valued for the positive correction parameter, which is not physical and therefore can be forbidden. We see that two critical points exist for the entropy density. The correction term affects significantly the entropy densities in between these critical points. Furthermore, we derive the first-order corrected Gibbs free energy density and discuss the effects of correction terms. We observe that the correction terms with negative correction parameter make Gibbs free energy density (more-)less negative valued for the (smaller-)larger black holes. However, the correction terms with positive correction parameter make Gibbs free energy density more positive valued for the black holes with smaller horizon radius. For the larger values of charge and AdS radius, the deviation of corrected Gibbs free energy density with their original value becomes less. We also calculated the corrected expression for the total charge of the quasitopological black holes which coincides with their original expression in limit $\alpha \to 0$. Moreover, we evaluate the first-order corrected expression for the mass density of this black hole. We find that a critical point exists for total mass density below which corrected terms with the positive correction parameter shows opposite behavior. We also check the stability and bound point of black holes by calculating specific heat at constant chemical potential and plot with respect to horizon radius. We find that the phase transition does not occur due to the correction term with positive correction parameter and black holes are in stable state. The correction term with negative parameter causes instability for such black holes. Furthermore, in the same fashion, we investigate the effects of thermal fluctuation on the thermodynamics of charged quasitopological black holes endowed with global rotation.

The paper is organized as follows. In section 2, we derive the corrected expression for entropy density due to the thermal fluctuations when the size of the black holes is reduced to the Planck scale. In section 3, we discuss the effects of quantum corrections due to thermal fluctuations on the thermodynamics of charged quasitopological black holes. Within this section, we study the influence of leading-order correction on stability of such black holes. In section 4, we consider a charged topological black holes endowed with global rotation and discuss the effects of thermal fluctuations on the thermodynamics of it. We also study the stability and bound points of charged rotating quasitopological black holes under the influence of thermal fluctuations. We summarize our results with concluding remarks in the last section 5.

2. Thermodynamics under (quantum) thermal instability: Preliminaries

In this section, we review the corrections to thermodynamic entropy density of the quasitopological black holes when small stable fluctuations around equilibrium are taken into account. In this connection, one may assume that the system of quasitopological black holes is characterized by the canonical ensemble. In order to begin the analysis, let us first define the density of states with fixed energy as [27,28]

$$\rho(E) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\mathcal{S}(\beta)} d\beta. \tag{1}$$

Here $\mathcal{S}(\beta)$ refers to the exact entropy density which is not just its value at equilibrium and depends on temperature $T = 1/\beta$ explicitly. The exact entropy density corresponds to the sum of entropy densities of subsystems of the thermodynamical system, which are small enough to be considered in equilibrium. In order to solve the complex integral (1), we utilize the method of steepest descent around the saddle point $\beta_0 (= 1/T_H)$ such that $\left(\frac{\partial \mathcal{S}(\beta)}{\partial \beta}\right)_{\beta=\beta_0} = 0$. We assume that the quasitopological black hole is in equilibrium at Hawking temperature T_H. Now, the Taylor expansion of exact entropy density around the saddle point $\beta = \beta_0$ yields

$$\mathcal{S}(\beta) = S_0 + \frac{1}{2}(\beta - \beta_0)^2 \left(\frac{\partial^2 \mathcal{S}(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0} + (\text{higher order terms}), \tag{2}$$

where $S_0 = \mathcal{S}(\beta_0)$ refers the leading-order entropy density. Now, by plugging this $\mathcal{S}(\beta)$ (2) into (1), and solving integral by choosing $c = \beta_0$ for positive $\left(\frac{\partial^2 \mathcal{S}(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0}$ leads to [19]

$$\rho(E) = \frac{e^{S_0}}{\sqrt{2\pi \left(\frac{\partial^2 \mathcal{S}(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0}}}. \tag{3}$$

The logarithm of the above density of states yields the corrected microcanonical entropy density at equilibrium (obtained by incorporating small fluctuations around thermal equilibrium)

$$S = S_0 - \frac{1}{2}\log\left(\frac{\partial^2 \mathcal{S}(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0} + (\text{sub-leading terms}). \tag{4}$$

By considering the most general form of the exact entropy density, $\mathcal{S}(\beta)$, the form of $\left(\frac{\partial^2 \mathcal{S}(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0}$ can be determined. The generic expression for leading-order correction to Bekenstein–Hawking formula is calculated by [19,18]

$$S = S_0 + \alpha \ln(S_0 T_H^2), \tag{5}$$

where α is a (constant) correction parameter. One should note that we considered a general correction parameter α because this is not