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Massive spin-3/2 models in $D = 2 + 1$

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ABSTRACT

We have demonstrated that the fermionic part of the so called "New Topologically Massive Supergravity", which is of third order in derivatives, is classically equivalent to self-dual models of lower order in derivatives, forming then a sequence of self-dual descriptions $SD(i)$, with $i = 1, 2, 3$ meaning the order in derivatives of each description. We have connected all the models by symmetry arguments through a Noether Gauge Embedment approach. This is completely equivalent to what happens in the bosonic cases of spins 1, 2 and 3, so some discussion about the similarities is made along the work. An analogue version of the Fierz–Pauli theory is suggested. Then through the NGE approach a fourth order model is obtained, which in our point of view would be the analogue version of the linearized New Massive Gravity theory for spin-3/2 fermionic particles.

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1. Introduction

The so called self-dual models have equivalent equations of motion, such equivalence persists even at the quantum level when we are able in to construct a master action which interpolate among the alternative descriptions. In general they differ each other by an order in derivatives and invariance under gauge transformations. The simplest example we can give refers to the well known equivalence between the self-dual SD [1] and the Maxwell–Chern–Simons models MCS [2]. In this case both of them describe a single massive spin-1 particle in $D = 2 + 1$ dimensions. Notice that the MCS model is gauge invariant while the self-dual is not, due to the presence of a Proca mass term.

Once such alternative descriptions also occurs for particles with higher spins, in the last years we have generalized the proof of equivalence we have observed in the case of spin-1 for spin-2 [3,4] and spin-3 [5,6] particles in $D = 2 + 1$. In particular we have learned with these bosonic examples that beyond the self-dual models which already exists one can find new models by means of systematic dualization procedures. Indeed in the case of spin-2 particles a parallel interest is in game. We have noticed that this dualization procedures can take us to the linearized versions of gravitational models. As an example the linearized version of the New Massive Gravity NMG model [7] can be obtained by mean of a generalized soldering of self-dual models [8,9].

In this work we have used the Noether Gauge Embedment NGE approach in order to establish the equivalence among self-dual models which describes massive spin-3/2 particles in $D = 2 + 1$ dimensions. In our initial scenario we have three self-dual descriptions of first $SD(1)$, second $SD(2)$ [10,11] and third order $SD(3)$ [12] in derivatives. The $SD(1)$ and the $SD(2)$ are already connected via a master action [10]. Here we will show that they can be connected by mean of symmetry arguments obtaining the second one by gauge embedding the first one. By the same technique we show that the $SD(3)$ can also be obtained in a second round of NGE connecting it with the previous ones. As in the spin-2 case we have also a parallel interest. In fact the $SD(3)$ model we have reached here consists of the fermionic part of the so called "New Topologically Massive Supergravity" NTMS. This may possibly suggest us new ways of understanding the building blocks of supergravity theories.

The self-dual models describes only a single propagation of helicity $+3/2$ or $-3/2$. In the fourth section of this work we have suggested a dublet model which describes both helicities $\pm 3/2$. We study the Fierz–Pauli conditions to this model in order to guarantee that it propagates only two degrees of freedom. The model we suggest is analogue to the Fierz–Pauli model, so it is non gauge invariant due to the presence of a mass term. Applying the NGE approach we obtain a fourth order model with same particle content, which seems to be an analogue version of the NMG theory at the linearized level. As well as the spin-2 particles have special appeal due to the fact they are closely related with gravitational

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models, here we think that the supergravity extension can also be considered in the future.

We start the next section by verifying the Fierz–Pauli conditions to the vector-spinor, counting then the number of degrees of freedom of the first-order self-dual model.

2. The first order self-dual model

The first order self-dual model introduced in [10] is given by:

$$S_{SD(1)} = \int d^3x \left(-\epsilon^{\mu\nu\alpha}\bar{\psi}_\mu\partial_\nu\psi_\alpha + m\epsilon^{\mu\nu\alpha}\bar{\psi}_\mu\gamma_\nu\psi_\alpha\right), \tag{1}$$

where the fields ψ_μ and $\bar{\psi}_\mu$ are Majorana two component vector-spinors. The Greek indices corresponds to the space–time components, while the spinorial indices has been suppressed. Then, it can be concluded that each vector-spinor field has six independent components in three space–time dimensions. The gamma matrices are indeed the Pauli matrices in agreement with [2], satisfying $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$ and $\gamma_\mu\gamma_\nu = \eta_{\mu\nu} + \epsilon_{\mu\nu\alpha}\gamma^\alpha$. Our metric is mostly plus $(-,+,+)$, and additional properties will be given along the work. About the model, we notice that it is quite similar to the bosonic cases of spins 1 [1], 2 [13] and 3 [14]. As in those models, the Chern–Simons like term is invariant under the gauge symmetries $\delta\psi_\mu = \partial_\mu\Lambda$ and $\delta\bar{\psi}_\mu = \partial_\mu\bar{\Lambda}$ where the $\Lambda's$ are arbitrary spinor fields. The same symmetries are broken by the mass term, and this suggest us to implement the NGE approach. However, first we would like to check that the model (1) satisfy all the Fierz–Pauli conditions i.e., from the equations of motion the field must be gamma-traceless $\gamma_\mu\psi^\mu = 0$ transverse $\partial_\mu\psi^\mu = 0$ and we have to be able to obtain a Klein–Gordon equation. All we have for the field ψ_μ one can demonstrate to the self-adjoint field, then let us consider the equations of motion with respect to the self-adjoint field:

$$\epsilon^{\mu\nu\alpha}\partial_\nu\psi_\alpha - m\epsilon^{\mu\nu\alpha}\gamma_\nu\psi_\alpha = 0. \tag{2}$$

One can notice that once the gamma matrices are constants, by applying ∂_μ on (2) we have $\epsilon^{\mu\nu\alpha}\gamma_\nu\partial_\mu\psi_\alpha = 0$. With this result in hand one can verify two Fierz–Pauli conditions. First notice that after applying γ_μ on (2) and using some gamma properties we can have $\gamma_\mu\psi^\mu = 0$. Then by rewriting $\epsilon^{\mu\nu\alpha}\gamma_\nu$ and using the fact that the field is now gamma-traceless we can demonstrate that it is also transverse $\partial^\mu\psi_\mu = 0$.

By multiplying the equation (2) by $\epsilon_{\mu\lambda\sigma}$ we have obtained:

$$-\partial_\lambda\psi_\sigma + \partial_\sigma\psi_\lambda + m\gamma_\lambda\psi_\sigma - m\gamma_\sigma\psi_\lambda = 0. \tag{3}$$

From here, we take ∂^λ on (3), which leaves us with $\Box\psi_\sigma - m\gamma_\lambda\partial^\lambda\psi_\sigma = 0$. On the other hand by applying γ^λ on (3) one obtain the Majorana equation:

$$(\gamma_\lambda\partial^\lambda - m)\psi_\sigma = 0. \tag{4}$$

With all together we have obtained the Klein–Gordon equation:

$$(\Box - m^2)\psi_\sigma = 0, \tag{5}$$

which finally completes the Fierz–Pauli conditions. This set of "constraints" are telling us that from the six independent components we left with only two of them. In order to have only one degree of freedom, we need one more constraint which indeed exists and tell us about the spin. This is the Pauli–Lubanski condition and we have to be able in deriving it from the equations of motion. To construct the Pauli–Lubanski operator we have to find out a spin-generator for spin-3/2 particles, which for example can be adapted from the reference [15], where the authors have provided an expression to the spin-generator in $D = 3+1$ dimensions which is an antisymmetric tensor $S_{\mu\nu}$. We can now adapt that result by noticing that in $D = 2+1$ dimensions any antisymmetric tensor can be rewritten as $S_{\mu\nu} = \epsilon_{\mu\nu\alpha}S^\alpha$, where S^α is precisely the object we are looking for. In this case we have:

$$(S^\alpha)^{\mu\nu} = i\epsilon^{\mu\alpha\nu} + i\frac{\gamma^\alpha}{2}\eta^{\mu\nu}. \tag{6}$$

Where we have verified that the spin generator for the spin-3/2 particle is in fact the sum of the spin-1 generator with the spin-1/2 generator, both given by [16]. Once we know that $P_\mu = i\partial_\mu$, it is possible to show that with the help of the Majorana equation (4), the equation of motion (2) can be rewritten as:

$$[(P.S)^{\mu\nu} + sm\eta^{\mu\nu}]\tilde{\psi}_\nu = 0, \tag{7}$$

where s is precisely 3/2. Notice that we have used a tilde variable, which means that this equation is valid only for the transverse and gamma-traceless field. Besides, it is straightforward to demonstrate that S_μ satisfy a Lee algebra since it is the sum of the spin-1 and 1/2 parts.

In the next section, we give symmetry arguments to find out a second-order self-dual model from the first order one, and then a third order-self-dual model from the second one.

3. The Noether Gauge Embedment – singlets

3.1. From $SD(1)$ to $SD(2)$

Since the first order self-dual model is non gauge invariant under $\delta\psi_\mu = \partial_\mu\Lambda$ and $\delta\bar{\psi}_\mu = \partial_\mu\bar{\Lambda}$, we may use the NGE approach to turn it gauge invariant. In consequence, by making that, the gauge invariant model becomes second-order in derivative. At the end of this section we would like to be able to establish the classical equivalence between the first and the second order self-dual models. In order to observe such equivalence, we are going to add source terms j_μ and $\bar{j}_\mu$ that will give us a dual map between the models. Then we start by rewritten (1) as:

$$S_{SD(1)} = \int d^3x \left(-\epsilon^{\mu\nu\alpha}\bar{\psi}_\mu\partial_\nu\psi_\alpha + m\epsilon^{\mu\nu\alpha}\bar{\psi}_\mu\gamma_\nu\psi_\alpha + \bar{\psi}_\mu j^\mu + \bar{j}^\mu\psi_\mu\right). \tag{8}$$

From (8) we obtain the Euler vector-spinors:

$$K^\mu = -\epsilon^{\mu\nu\alpha}\partial_\nu\psi_\alpha + m\epsilon^{\mu\nu\alpha}\gamma_\nu\psi_\alpha + j^\mu \tag{9}$$

$$\bar{K}^\mu = -\epsilon^{\mu\nu\alpha}\partial_\nu\bar{\psi}_\alpha + m\epsilon^{\mu\nu\alpha}\bar{\psi}_\nu\gamma_\alpha + \bar{j}^\mu, \tag{10}$$

and introduce a first-iterated action given by:

$$S_1 = S_{SD(1)} + \int d^3x \left(\bar{a}_\mu K^\mu + \bar{K}^\mu a_\mu\right), \tag{11}$$

where $\bar{a}_\mu$ and a_μ are auxiliary fields. By taking the gauge variation of (11) and choosing properly $\delta a_\mu = -\partial_\mu\Lambda$ and $\delta\bar{a}_\mu = -\partial_\mu\bar{\Lambda}$ it is straightforward to demonstrate that we can have:

$$\delta_{\Lambda,\bar{\Lambda}}S_1 = \int d^3x\, \delta\left(-m\epsilon^{\mu\nu\alpha}\bar{a}_\mu\gamma_\nu a_\alpha\right). \tag{12}$$

Automatically from (12) we have a gauge invariant model given by:

$$S_2 = S_{SD(1)} + \int d^3x \left(\bar{a}_\mu K^\mu + \bar{K}^\mu a_\mu + m\epsilon^{\mu\nu\alpha}\bar{a}_\mu\gamma_\nu a_\alpha\right). \tag{13}$$

Then by eliminating the auxiliary fields with the help of their equations of motion we have after some manipulation:

$$a^\mu = -\frac{\gamma_\nu\gamma^\mu K^\nu}{2m} \quad ; \quad \bar{a}^\mu = -\frac{\bar{K}^\nu\gamma^\mu\gamma_\nu}{2m}, \tag{14}$$

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