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Deriving three-dimensional bosonization and the duality web

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ABSTRACT

Recently, a duality web for three dimensional theories with Chern–Simons terms was proposed. This can be derived from a single bosonization type duality, for which various supporting arguments (but not a proof) were given. Here we explicitly derive this bosonization, in the Abelian case and for a particular regime of parameters. To do this, we use the particle-vortex duality in combination with a Buscher-like duality (both considered in the regime of low energies). As a corollary, Son's conjectured duality is derived in a somewhat singular limit of vanishing mass.

1. Introduction

Duality symmetries are powerful tools that serve to constrain and understand non-perturbative physics. In $(2+1)$-dimensions, within the context of condensed matter systems, dualities have received less attention in comparison to their $(3+1)$-dimensional counterparts, that naturally appear in particle physics. Recently however, partly motivated by the desire to understand Son's conjecture [1] a web of dualities was proposed [2–4].

In fact, D.T. Son has proposed a relation between a massless Dirac 'fundamental' fermion and a 'composite' Dirac fermion coupled to a gauge field with BF-dynamics [1]. The 'fundamental' fermion is to be understood as a boundary mode in a topological insulator, while the 'composite' one should be thought of as an effective description for a half-filled lowest Landau level of a Fermi liquid [1], [5]. The whole idea is driven by the field theoretical descriptions of a (time reversal invariant) topological insulator and a topological superconductor.

The web of dualities mentioned above relates various bosonic theories (for scalars and gauge fields) with fermionic theories (coupled to a vector field), both with Chern–Simons terms. All fields transform under a $U(1)$ gauge symmetry. Extensions, including to non-Abelian cases have been considered in [6–10].

The web of dualities can be derived by assuming the validity of a basic correspondence between a bosonic theory and a fermionic one, which in the rest of this paper will be referred to as *three-dimensional bosonization*. These ideas were considered and extended to the context of supersymmetric theories by Aharony [11].

On the other hand, an explicit mapping between a bosonic and a fermionic theory was presented around twenty years ago in [12], [13,14]. It is based on the realisation that two-dimensional bosonization can be viewed as a Buscher-like duality and the extension of such procedure to three dimensions. We will refer to it as the *Burgess–Quevedo map* (or BQ-map). See the paper [15] for a careful account of the idea and technical details of the BQ-map.

Postulating the validity of the three-dimensional bosonization duality, one can derive the (bosonic) particle-vortex duality, or the fermionic duality conjectured by Son. Repeated application leads to a full duality web. While this basic three-dimensional bosonization was not proven, evidence indicative of its correctness was presented in [3,4] and subsequent papers, e.g. [16].

In this note, we will derive this basic three-dimensional bosonization conjecture as presented in [4,7]. To do this, we will assume the validity of the particle-vortex duality and combine it with the Buscher-like BQ-map [12], [13], [14]. In fact, a regime of sufficiently low energies, with special field configurations in the particle-vortex equivalence, together with a BQ-map improved by the presence of point like vortices in the system, are instrumental to our derivation.

Our approach will be phrased in the framework of the path integral formalism, as defined, for example, in [17], which is itself based on the earlier work [18] (see also [19,20] for an alternative viewpoint and [21] for the usual condensed matter formulation). The particle-vortex duality was discussed in works on supercon-

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ductivity [22], [23,24], and also in the contexts of anyon superconductivity and the fractional quantum Hall effect [25].

This work is organized as follows. In Section 2, we summarize and streamline the background material needed for our purposes: the three-dimensional bosonization proposal, time reversal, Son's duality and the particle vortex duality. In Section 3 we derive the conjectured three dimensional bosonization, assuming the validity of the particle-vortex duality and the BQ-map. Section 4 closes the paper with final conclusions.

2. Three-dimensional bosonization and the duality web

As a warm-up, in this section we will review how (part of) the Abelian duality web is derived. We will also discuss the action of time reversal on the different dualities and go over the derivation in [3,4] of Son's conjectured relation [1]. Finally, the particle-vortex duality will be shown to arise from alternate integrations on a 'master' partition function that depends on both 'particle' and 'vortex' fields.

A main basic ingredient in this work is the three dimensional bosonization that we now review, adopting the notation in [4]. The partition functions for a complex scalar $\phi = \phi_0 e^{i\theta}$ coupled to a vector A_μ (adding 'flux'), and that for a Dirac fermion ψ (both in the presence of a vectorial external source S_μ) are

$$\begin{aligned}\tilde{Z}_{\text{scalar+flux}}[S] &\equiv \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}A_\mu e^{iS_{\text{scalar}}[\phi,A]+iS_{\text{CS}}[A]+iS_{\text{BF}}[A,S]}\\ &= \int \mathcal{D}\phi_0\mathcal{D}\theta\mathcal{D}A_\mu\\ &\quad e^{iS_{\text{scalar}}[\theta,A;\phi_0]-\frac{1}{2}\int d^3x(\partial_\mu\phi_0)^2+iS_{\text{CS}}[A]+iS_{\text{BF}}[A,S]}\\ Z_{\text{fermion}}[S;m] &\equiv \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{i\int\bar{\psi}(\not{\partial}+m+\not{S})\psi}. \end{aligned}\tag{2.1}$$

We have denoted,

$$\begin{aligned}S_{\text{CS}}[A] &\equiv \frac{1}{4\pi}\int d^3x\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho,\\ S_{\text{BF}}[A,S] &\equiv \frac{1}{2\pi}\int d^3x\epsilon^{\mu\nu\rho}A_\mu\partial_\nu S_\rho.\end{aligned}\tag{2.2}$$

The action for the complex scalar ϕ is defined and can be rewritten according to,

$$\begin{aligned}S_{\text{scalar}}[\phi,A] &\equiv -\frac{1}{2}\int d^3x|(\partial_\mu - iA_\mu)\phi|^2 \to S_{\text{scalar}}[\theta,A;\phi_0]\\ &\equiv -\frac{1}{2}\int d^3x\phi_0^2(\partial_\mu\theta + A_\mu)^2.\end{aligned}\tag{2.3}$$

Note that the scalar action $S_{\text{scalar}}[\theta, A;\phi_0]$ in the last expression of eq. (2.3) appears for the case in which the modulus ϕ_0 is constrained to be constant. Such an action is obtained from that of a complex scalar ϕ with a symmetry breaking Higgs-like potential,

$$\begin{aligned}S_{\text{scalar}}[\theta,A;\phi_0] &= \lim_{\alpha\to\infty} S_{\text{scalar}}[\theta,A;\phi_0]\\ &\quad -\int d^3x\left[\frac{1}{2}(\partial_\mu\phi_0)^2 + \frac{\alpha}{2}(\phi_0^2-m)^2\right]\\ &= \lim_{\alpha\to\infty} S_{\text{scalar}}[\phi,A] - \int d^3x\frac{\alpha}{2}(\phi_0^2-m)^2,\end{aligned}\tag{2.4}$$

with the coupling α taken to be very large, $\alpha\to\infty$. Equivalently, for low energies $E \ll \alpha$, the quantity ϕ_0 takes a constant value. In most of the analysis below we will consider ϕ_0 to be fixed, $\phi_0 = \sqrt{m}$, and we will drop $\int\mathcal{D}\phi_0$ from the path integral.

Then, the basic three-dimensional bosonization duality, relates a fermion coupled to a background vectorial current with a complex scalar plus flux, considered in general with a fluctuating ϕ_0 (hence the tilde on $Z_{\text{scalar+flux}}$). More explicitly,

$$Z_{\text{fermion}}[S;m=0]e^{-\frac{i}{2}S_{\text{CS}}[S]} = \tilde{Z}_{\text{scalar+flux}}[S].\tag{2.5}$$

In the paper [7], the authors proposed a more general duality for the bosonization of a massive fermion (of mass m). This extended relation that leads to a more general web of dualities reads

$$\begin{aligned}&Z_{\text{fermion}}[S;m]e^{-\frac{i}{2}S_{\text{CS}}[S]} = Z_{\text{scalar+flux}}[S].\\ &Z_{\text{scalar+flux}}[S] = \lim_{\alpha\to\infty, E\ll\alpha}\int\mathcal{D}A_\mu\mathcal{D}\phi_0\mathcal{D}\theta\mathcal{D}\sigma\\ &\qquad e^{iS_{\text{scalar}}(\theta,A;\phi_0)+iS_{\text{CS}}[A]+iS_{\text{BF}}[A,S]-i\int d^3x\left[\frac{1}{2}(\partial_\mu\phi_0)^2+\sigma(\phi_0^2-m)+\frac{\sigma^2}{2\alpha}\right]}.\end{aligned}\tag{2.6}$$

In the case of vanishing mass ($m = 0$), integrating out the non-dynamical field σ we generate a potential $V = \phi_0^4/2\alpha$, which leads to the Wilson–Fisher fixed point at low energies. If $m > 0$, integrating out σ we get the Higgs-like potential in eq. (2.4). At small enough energies $E \ll m = \phi_0^2$, $E \ll \alpha$, the dynamical field ϕ_0 freezes-out, leaving us simply with $Z_{\text{scalar+flux}}[S]$ on the right hand side (notice that the integration in ϕ_0 is trivial, hence the absence of tilde in $Z_{\text{scalar+flux}}[S]$). More explicitly, at low energies and after the constraint is imposed, we have

$$Z_{\text{scalar+flux}}[S] = \int\mathcal{D}A_\mu\mathcal{D}\theta e^{iS_{\text{scalar}}(\theta,A;\phi_0)+iS_{\text{CS}}[A]+iS_{\text{BF}}[A,S]}.\tag{2.7}$$

In the following we will consider the situation in which the constraint $\phi_0^2 = m$ is enforced by the integration over the field σ, in the limit of low energies. More precisely, we will probe the dynamics with energies that are very small compared to those set by the two relevant scales, m and α.

2.1. Time-reversed relation

Another ingredient needed to prove different entries of the duality web comes from considering the effect of time reversal on the system. Time reversal invariance leads to relations, which change the sign of the Chern–Simons and BF terms. Indeed, we also have the duality,

$$\begin{aligned}Z_{\text{fermion}}[S]e^{+\frac{i}{2}S_{\text{CS}}[S]} &= \bar{\tilde{Z}}_{\text{scalar+flux}}[S]\\ &\equiv\int\mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}A_\mu e^{iS_{\text{scalar}}[\phi,A]-iS_{\text{CS}}[A]-iS_{\text{BF}}[A,S]}.\end{aligned}\tag{2.8}$$

The bosonic and fermionic particle-vortex dualities are obtained by applying and manipulating the three-dimensional bosonization relation in eq. (2.5), and using then the time-reversed bosonization relation above.

2.2. Son's duality from bosonization

As an example, we derive Son's conjectured duality between a massless Dirac fermion ψ coupled to an external field S_μ and a composite Dirac fermion χ, coupled to a dynamical field A_μ, which itself couples to the external S_μ through a BF coupling, denoted BF-QED. In what follows, we summarise a derivation in [3], [4]. Indeed, the dynamics of the composite fermion χ and the vector A_μ is described by

$$Z_{\text{BF-QED}}[S;m] = \int\mathcal{D}A_\mu\mathcal{D}\chi\mathcal{D}\bar{\chi}e^{i\int\bar{\chi}(\not{\partial}+m+\not{A})\chi+\frac{i}{2}S_{\text{BF}}[A,S]}.\tag{2.9}$$