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String scattering amplitudes and deformed cubic string field theory

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ABSTRACT

We study string scattering amplitudes by using the deformed cubic string field theory which is equivalent to the string field theory in the proper-time gauge. The four-string scattering amplitudes with three tachyons and an arbitrary string state are calculated. The string field theory yields the string scattering amplitudes evaluated on the world sheet of string scattering whereas the conventional method, based on the first quantized theory brings us the string scattering amplitudes defined on the upper half plane. For the highest spin states, generated by the primary operators, both calculations are in perfect agreement. In this case, the string scattering amplitudes are invariant under the conformal transformation, which maps the string world sheet onto the upper half plane. If the external string states are general massive states, generated by non-primary field operators, we need to take into account carefully the conformal transformation between the world sheet and the upper half plane. We show by an explicit calculation that the string scattering amplitudes calculated by using the deformed cubic string field theory transform into those of the first quantized theory on the upper half plane by the conformal transformation, generated by the Schwarz–Christoffel mapping.
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1. Introduction

String theory was formulated first in terms of the scattering amplitude, which is now known as Veneziano amplitude [1]. The Veneziano amplitude was soon generalized to the Virasoro–Shapiro amplitude [2,3]. These scattering amplitudes marked the birth of the string theory. The Veneziano amplitude corresponds to the four-tachyon scattering amplitude of open string, and the Virasoro–Shapiro scattering amplitude corresponds to the four-tachyon scattering amplitude of closed string. Since the birth of string theory, the string scattering amplitude has played a major role in the development of string theory. The string scattering amplitudes usually have been studied by using the first quantized string theory. In the first quantized theory, we may obtain the string scattering amplitudes by evaluating the Polyakov string path integral with vertex operators inserted on the string world sheet. For open string theory, the upper half plane is chosen as the string world sheet and the vertex operators, representing external string states, are inserted on the real axis. The string scattering amplitudes are found to enjoy various relationships between themselves, which are important in understanding the profound nature of string theory. Among them are string Bern–Carrasco–Johansson (BCJ) relations [4–7], which are the string theory generalizations of gauge field theory BCJ relations [8] and the Kawai–Lewellen–Tye (KLT) relations [9], which relate the tree level closed string scattering amplitudes to those of open string. The string scattering amplitudes also satisfy the generalized on-shell Ward identities [10–12] which, together with the string BCJ relation, drastically reduce the number of independent string scattering amplitudes.

In the present work, we shall study the string scattering amplitudes in the framework of the covariant interacting string field theory. In particular, by using the deformed cubic string field theory [13], we will calculate the four-string scattering amplitudes with three tachyons and an arbitrary string states, which have been extensively studied in Refs. [14–21] to explore the symmetric properties of the string scattering amplitudes in the high energy limit [22–26]. In recent works [13,27], one of the authors showed that the non-planar world sheets of Witten's cubic open string field theory [28,29] can be made planar if we choose the external string states judiciously. The deformed cubic string field

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theory, which is equivalent to the covariant string field theory in the proper-time gauge [30], has a number of advantages over other approaches: 1) The world sheet diagrams of the string scattering are planar so that we can apply the light-cone string field theory techniques [31–37]. 2) The theory does not contain unphysical length parameters [38,39] and yet possesses the BRST gauge symmetry because its action is formally equivalent to that of Witten's cubic string field theory [28]. 3) We can obtain the exact gauge invariant Yang–Mills field action without using the level truncations [40] or the field redefinitions [41] in the zero-slope limit. We expect that the deformed cubic string field theory or the covariant string field theory in the proper-time gauge produces the string scattering amplitudes which can be directly compared with those of the first quantized string theory.

2. String scattering amplitudes

We shall begin with a brief review of the recent work [19] on the string scattering amplitudes (SSA) of three tachyons and one arbitrary string states which are based on the first quantized string theory. It fixes the notations and the kinematics of the scattering in the center of mass frame. In the center of mass frame, the kinematics are defined as

$$k_1 = \left(\sqrt{M_1^2 + |\vec{k_1}|^2}, -|\vec{k_1}|, 0\right), \tag{1a}$$

$$k_2 = \left(\sqrt{M_2 + |\vec{k_1}|^2}, +|\vec{k_1}|, 0\right), \tag{1b}$$

$$k_3 = \left(-\sqrt{M_3^2 + |\vec{k_3}|^2}, -|\vec{k_3}|\cos\phi, -|\vec{k_3}|\sin\phi\right), \tag{1c}$$

$$k_4 = \left(-\sqrt{M_4^2 + |\vec{k_3}|^2}, +|\vec{k_3}|\cos\phi, +|\vec{k_3}|\sin\phi\right) \tag{1d}$$

with $M_1^2 = M_3^2 = M_4^2 = -2$ and ϕ is the scattering angle. The Mandelstam variables are defined as

$$s = -(k_1+k_2)^2, \quad t = -(k_2+k_3)^2, \quad u = -(k_1+k_3)^2. \tag{2}$$

We choose the three polarization vectors on the scattering plane as follows:

$$e^T = (0,0,1), \tag{3a}$$

$$e^L = \frac{1}{M_2}\left(|\vec{k_1}|, \sqrt{M_2 + |\vec{k_1}|^2}, 0\right), \tag{3b}$$

$$e^P = \frac{1}{M_2}\left(\sqrt{M_2 + |\vec{k_1}|^2}, |\vec{k_1}|, 0\right). \tag{3c}$$

Note that SSA of three tachyons and one arbitrary string state with polarizations orthogonal to the scattering plane vanish. For later use, we define

$$k_i^X \equiv e^X \cdot k_i \ \text{ for } \ X = (T, P, L). \tag{4}$$

The general string states at mass level

$$M_2^2 = 2(N-1), \quad N = \sum_{n,m,l>0}\left(n r_n^T + m r_m^P + l r_l^L\right), \tag{5}$$

with polarizations on the scattering plane are of the form

$$\left|r_n^T, r_m^P, r_l^L\right\rangle = \prod_{n>0}\left(\alpha_{-n}^T\right)^{r_n^T}\prod_{m>0}\left(\alpha_{-m}^P\right)^{r_m^P}\prod_{l>0}\left(\alpha_{-l}^L\right)^{r_l^L}|0,k\rangle. \tag{6}$$

The four-string scattering amplitude with three tachyons and one general string state Eq. (6) in the s-t channel is found to be [19]

$$\begin{aligned} A_{st}^{(r_n^T;r_m^P;r_l^L)} &= \int_0^1 dx\, x^{k_1\cdot k_2}(1-x)^{k_2\cdot k_3} \\ &\prod_{n=1}\left\{(-1)^{n-1}(n-1)!\left(\frac{k_1^T}{x^n} + \frac{k_3^T}{(x-1)^n}\right)\right\}^{r_n^T} \\ &\prod_{m=1}\left\{(-1)^{m-1}(m-1)!\left(\frac{k_1^P}{x^m} + \frac{k_3^P}{(x-1)^m}\right)\right\}^{r_m^P} \\ &\prod_{l=1}\left\{(-1)^{l-1}(l-1)!\left(\frac{k_1^L}{x^l} + \frac{k_3^L}{(x-1)^l}\right)\right\}^{r_l^L}. \end{aligned} \tag{7}$$

Here the four points on the real line where the vertex operators are inserted, are chosen as

$$Z_1 = 0, \quad Z_2 = x, \quad Z_3 = 1, \quad Z_4 = \infty. \tag{8}$$

In the s-t channel, the Koba–Nielsen variable x is in the range of $[0,1]$ while in the t-u channel, x is in the range of $[1,\infty)$. The four-string scattering amplitude is related to that of s-t channel through the string BCJ relation:

$$\begin{aligned} A_{st}^{(r_n^T;r_m^P;r_l^L)} &= \frac{\sin(\pi k_2\cdot k_4)}{\sin(\pi k_1\cdot k_2)} A_{tu}^{(r_n^T;r_m^P;r_l^L)} \\ &= \frac{\sin(\frac{u}{2}+2-N)\pi}{\sin(\frac{s}{2}+2-N)\pi} A_{tu}^{(r_n^T;r_m^P;r_l^L)}. \end{aligned} \tag{9}$$

We can prove this string BCJ relation by an explicit calculation [19], rewriting the SSA in the s-t channel in terms of the D-type Lauricella function $F_D^{(K)}$ which is one the four extensions of the Gauss hypergeometric function. The exact calculation of SSA in terms of the Lauricella function was useful to redrive the symmetric relations among SSA in various limits: These include the linear relations in the hard scattering limit [11,12,44], the recurrence relations in the Regge scattering limit [17,42,43], and the extended recurrence relations in the non-relativistic scattering limit [7]. Reader may refer to Ref. [12] for more details.

3. Deformed cubic string field theory

All types of symmetric properties of string theory should also be deduced from the second quantized theory, i.e., the covariant interacting string field theory. The string field theory may provide us a more coherent framework to understand the symmetric properties of string theory at a deeper level. However, in practice, it has been difficult to make use of the string field theory to calculate the SSA. The major obstacle was the non-planarity of the world sheet diagrams of the cubic open string field theory. The world sheet of N-string vertex is a conic surface with an excess angle of $(N-2)\pi$. The Fock space representation of three-string vertex has been obtained by Gross and Jevicki [45,46] by mapping the world sheet of six strings onto a circular disk with an orbifold condition and that of the four-string vertex has been constructed by Giddings [47] by mapping the world sheet of four-string vertex onto the upper half plane with branch cuts. However, it is hard to make use of these constructions to calculate the SSA of four strings. It is so complicated to compute the Neumann functions for the four-string vertex by using the conformal mapping given in Ref. [47]. One may obtain the four-string vertex from the cubic string field action by using the Fock space representation of the three-string vertex by Gross and Jevicki as an effective interaction. However, this procedure may involve inverting infinite dimensional matrices and does not yield exact results.

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