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Anatomy of the magnetic catalysis by renormalization-group method

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A B S T R A C T

We first examine the scaling argument for a renormalization-group (RG) analysis applied to a system subject to the dimensional reduction in strong magnetic fields, and discuss the fact that a four-Fermi operator of the low-energy excitations is marginal irrespective of the strength of the coupling constant in underlying theories. We then construct a scale-dependent effective four-Fermi interaction as a result of screened photon exchanges at weak coupling, and establish the RG method appropriately including the screening effect, in which the RG evolution from ultraviolet to infrared scales is separated into two stages by the screening-mass scale. Based on a precise agreement between the dynamical mass gaps obtained from the solutions of the RG and Schwinger–Dyson equations, we discuss an equivalence between these two approaches. Focusing on QED and Nambu–Jona-Lasinio model, we clarify how the properties of the interactions manifest themselves in the mass gap, and point out an importance of respecting the intrinsic energy-scale dependences in underlying theories for the determination of the mass gap. These studies are expected to be useful for a diagnosis of the magnetic catalysis in QCD.

1. Introduction

Strong magnetic fields confine charged fermions in the lowest Landau levels (LLLs), and they enjoy the properties of the $(1+1)$-dimensional chiral fermions with the dispersion relation $[e\boldsymbol{B} = (0, 0, eB),\ eB > 0]$:

$$\epsilon_{\rm LLL}^{R/L} = \pm p_z\,. \tag{1}$$

Intuitively, this is a consequence of the formation of the small cyclotron orbit with the radius $\sim 1/|eB|^{1/2}$ and the residual free motion along the field. It turned out that this *dimensional reduction* gives rise to rich physics phenomena. Especially, the *magnetic catalysis* of the chiral symmetry breaking and the *chiral magnetic effect* have been addressed by many authors (see, e.g., Refs. [1,2] and Refs. [3–6] for reviews).

The clear statement on the physical mechanism of the magnetic catalysis was due to Gusynin, Miransky, and Shovkovy in terms of a simple four-Fermi interaction [7]. By solving the gap equation of the NJL model, they found a mass gap

$$m_{\rm dyn} = \sqrt{eB}\,\exp\left(-\frac{\pi}{\rho_{\rm LLL} G_{\rm NJL}}\right), \tag{2}$$

where $\rho_{\rm LLL}$ and $G_{\rm NJL}$ are the density of states in the LLL and a dimensionful coupling constant of the four-Fermi interaction, respectively. Their core observation is seen in the similarity between the mass gap and the energy gap of superconductivity which is given by $\Delta \sim \omega_{\rm D} \exp[-c'/(\rho_{\rm F} G')]$ with $\omega_{\rm D}$ and $\rho_{\rm F}$ being the Debye frequency and the density of states near the Fermi surface, respectively. Also, G' and c' are a coupling constant and a positive number, respectively. In fact, this similarity is originated from the dimensional reduction in the low-energy domains of the both theories, i.e., in the LLL and in the vicinity of the Fermi surface.

We can clearly see the consequence of the dimensional reduction by focusing on QED in the weak coupling regime. From the rainbow approximation of the Schwinger–Dyson (SD) equation, the mass gap was obtained as

$$m_{\rm dyn} \simeq \sqrt{eB}\,\exp\left(-\frac{\pi}{2}\sqrt{\frac{\pi}{\alpha}}\right), \tag{3}$$

with an unscreened photon propagator in the early studies [8–10], and also

$$m_{\rm dyn} \simeq \sqrt{2eB}\,\alpha^{1/3}\exp\left\{-\frac{\pi}{\alpha\log(C\pi/\alpha)}\right\}, \tag{4}$$

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with a screened photon propagator [11,12]. Here, $\alpha = e^2/4\pi$ and C is a certain constant of order one. The constant was analytically obtained as $C = 1$ when the momentum dependence of $m_{\rm dyn}$ is neglected. The authors of Refs. [11,12] observed that *the gap equation always has a nontrivial solution irrespective of the size of the coupling constant*, indicating that the strong magnetic fields cause the dynamical symmetry breaking without support of any other nonperturbative dynamics. This reminds us of the well-known fact that any weak attractive interaction causes superconductivity.

Our main assertion in this Letter is that all these aspects of the magnetic catalysis can be understood with the Wilsonian renormalization group (RG) analysis. We will show that the emergence of the dynamical mass gap is informed from the RG flow for the effective four-Fermi operator that goes into the Landau pole. Bearing it in mind that four-Fermi operators are irrelevant in ordinary $(3+1)$-dimensional systems, we will clearly see from the RG point of view that the magnetic catalysis of the dynamical symmetry breaking is intimately related to the dimensional reduction. Our approach shares the philosophy with the analysis of (color) superconductivity by the RG method [13–20].

Our ultimate goal is to consistently understand the enhancement of the chiral symmetry breaking at zero or low temperature, and the inverse magnetic catalysis near the chiral phase transition temperature in QCD. There has been a discrepancy between the estimates of the chiral condensate from the lattice QCD simulation and typical model calculations [21]. For the magnetic catalysis and the inverse catalysis to be compatible with each other, it appeared to be important to explain a mechanism which makes the dynamical mass gap stay as small as the QCD scale $\Lambda_{\rm QCD}$ even in a strong magnetic field $eB \gg \Lambda_{\rm QCD}^2$ [22] (see also Refs. [23–25]). The method of renormalization group is a potentially useful tool to obtain a clear insight on this issue on the basis of an argument of the hierarchy which we will elaborate in the present paper.

However, to the best of our knowledge, even the correct form of the mass gap in weak-coupling gauge theories has not been obtained by the RG analyses in the presence of the screening effect. Therefore, before discussing the strong-coupling regime in QCD, one should understand how the screening effects are reflected in the parametric form of the mass gap in a clear way. Moreover, it is a generic issue to establish a systematic way of including the screening effects in the RG analyses, which will be important in a variety of systems. Note, for example, that there was an issue of the color magnetic screening in the RG analysis on the color superconductivity [17].

We will show that all of the results in Eqs. (2), (3), and (4) from the SD equations are precisely obtained from the solutions of the RG equations. Furthermore, we will clarify the origins of the overall factor of $\sqrt{eB}$ and the exponents in the language of the RG method. We will find that the properties of the interactions in the model/theory are directly reflected in the parametric dependences of the dynamical mass on the coupling constant and the magnitude of eB. Ultimately, these studies will be useful for a diagnosis of the magnetic catalysis in QCD. We will come back to this point with a brief comment on the perspective in the last section.

More specifically, we will closely look into the screening effect on the photon propagator. It would be instructive to mention a successful application of the RG method to color superconductivity in dense quark matter, where an appropriate treatment of the dynamical screening effect on the magnetic gluons was important for obtaining the correct magnitude of the gap [17,20]. We should also mention that the RG analysis of the magnetic catalysis at weak coupling was performed in Refs. [26,27]. Also, the magnetic catalysis in QCD was investigated on the basis of both the SD and RG equations in Ref. [28]. However, roles of the screening effect arising from the quark loop in the magnetic field have not been identified thus far, and we are not aware of the RG analysis in the literature of which the result agrees with that from the SD equation (4).

As we will discuss later in more detail, the screening effect should be appropriately incorporated in the derivation of the RG equation, since the screening mass sets an intrinsic energy scale of the underlying theory in between the ultraviolet and infrared regimes. The essential technique was recently developed for the analysis of the RG flow in "magnetically induced QCD Kondo effect" [29]. In the present Letter, we will show that the same technique successfully works for the analysis of the magnetic catalysis at weak coupling.

The structure of this Letter is the following. We first show the connection between the magnetic catalysis and the dimensional reduction which can be understood from a simple discussion of the scaling dimensions. Next, we construct an effective four-Fermi interaction from the underlying weak-coupling theory, i.e., QED, and appropriately include the energy-scale dependence of the tree-level interaction. Based on these discussions, we derive the RG equations and obtain the dynamical mass gap from their solutions. We confirm that the energy-scale dependence of the interaction is necessary for obtaining the correct form of the gap, which was however missing in the previous analyses. Finally, we discuss the correspondences between the RG and SD analyses, and the crucial roles of the photon/gluon propagators in the magnetic catalysis. The derivation of the RG equation is briefly summarized in an appendix.

2. Infrared scaling dimensions

We begin with looking into an analogy between the systems in the strong magnetic field and at high density. In the presence of a large Fermi sphere, the low-energy excitations near the Fermi surface show the dimensional reduction: The two-dimensional phase space tangential to the large Fermi sphere is degenerated, and the energy dispersion depends only on the momentum normal to the sphere. Then, the dimensional reduction enhances the infrared (IR) dynamics, leading to the instabilities near the Fermi surface. Based on the analogy with this mechanism, Gusynin et al. clearly pointed out that the chiral symmetry breaking occurs in the strong magnetic field no matter how weak the coupling is [7].

One can see possible emergence of the IR instability from a simple argument of the scaling dimensions. The kinetic term for the LLL reads

$$S_{\rm LLL}^{\rm kin} = \int dt \int dp_z \bar{\psi}_{\rm LLL}(p_z)(i\partial_t \gamma^0 - p_z\gamma^3)\psi_{\rm LLL}(p_z), \tag{5}$$

where we have suppressed the label specifying the location of the cyclotron center on the transverse plane. From this kinetic term, one can find the IR scaling dimension of the LLL fermion field when the excitation energy goes down toward zero as $\epsilon_{\rm LLL} \to s\epsilon_{\rm LLL}$ $(t \to s^{-1}t)$ with $s < 1$. Since the LLL fermion has the $(1+1)$-dimensional dispersion relation (1), the longitudinal momentum p_z also scales as $p_z \to sp_z$. On the other hand, the transverse momentum does not scale, because it serves just as the label of the degenerated states and does not appear in the dispersion relation (1). Therefore, when the kinetic term (5) is invariant under the scale transformation, the LLL fermion field scales as $s^{-1/2}$ in the low-energy dynamics.

Bearing this in mind, we proceed to the effective four-Fermi operator in the LLL:

$$S_{\rm LLL}^{\rm int} = \int dt \prod_{i=1,2,3,4} \int dp_z^{(i)}\, G\, \delta(p_z^{(1)} + p_z^{(2)} - p_z^{(3)} - p_z^{(4)}) \times \left[\bar{\psi}_{\rm LLL}(p_z^{(2)})\gamma_\parallel^\mu \psi_{\rm LLL}(p_z^{(4)})\right]\left[\bar{\psi}_{\rm LLL}(p_z^{(3)})\gamma_{\parallel\mu}\psi_{\rm LLL}(p_z^{(1)})\right], \tag{6}$$