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Degenerate vacua to vacuumless model and kink–antikink collisions

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ABSTRACT

In this work we investigate a Z_2 symmetric model of one scalar field ϕ in $(1, 1)$ dimension. The model is characterized by a continuous transition from a potential $V(\phi)$ with two vacua to the vacuumless case. The model has kink and antikink solutions that minimize energy. Stability analysis is described by a Schrödinger-like equation with a potential that transits from a volcano-shape with no vibrational states (in the case of vacuumless limit) to a smooth valley with one vibrational state. We are interested in the structure of two-bounce windows present in kink–antikink scattering processes. The standard mechanism of Campbell–Schonfeld–Wingate (CSW) requires the presence of one vibrational state for the occurrence of two-bounce windows. We report that the effect of increasing the separation of vacua from the potential $V(\phi)$ has the consequence of trading some of the first two-bounce windows predicted by the CSW mechanism by false two-bounce windows. Another consequence is the appearance of false two-bounce windows of zero-order.

1. Introduction

Solitary waves are solutions from nonlinear physics characterized by the very special property of localized density energy that can freely propagate without distortion in their form. The realization of solitary waves in nature is now amply recognizable, with interesting effects in solid state and atomic physics [1].

The simplest solitary wave is described by the $(1, 1)$ dimensional kink. Embedded in higher dimensions, the kinks generate domain walls or more generally p-branes in scenarios with extra dimensions. Brane collisions were considered as one possibility for generating the big bang, as in cyclic universe scenarios [2–4]. In particular, Refs. [3,4] considered particle production at the brane collision. For this, the simplest scenario of two domain walls in 5D Minkowski spacetime was considered. In the context of bubble collisions in the primordial universe, in some limit the effect of gravitational interaction between the bubbles can be negligible and the process of collisions can be described by kink–antikink in an effective $(1, 1)$ dimensional model [5–7]. In this way the collision of nearly planar walls is a necessary step in understanding the full nonlinear dynamics of collisions between pairs of bubbles nucleating in false vacuum [8].

Usually kink models are constructed starting from a Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi), \quad (1)$$

where the ϕ is a real scalar field and a potential $V(\phi)$ with minima at non-zero values of ϕ that suffers a spontaneous symmetry breaking. The equation of motion for the field $\phi(x, t)$ is given by

$$\phi_{tt} - \phi_{xx} + V_\phi = 0, \quad (2)$$

where we use the compact notation $V_\phi \equiv dV/d\phi$ and the like (similarly for higher-order derivatives, $V_{\phi\phi} \equiv d^2V/d\phi^2$). If we can write the potential in terms of the superpotential $V(\phi) = \frac{1}{2}W_\phi^2$, static configurations obeys one of the two first-order ordinary differential equations

$$\frac{d\phi}{dx} = \pm W_\phi. \quad (3)$$

The defects formed with this prescription minimize energy and are known as BPS defects [9,10].

Stability analysis considers small fluctuations

$$\Phi(x, t) = \phi(x) + \eta(x)\cos(\omega t), \quad (4)$$

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resulting in a Schrödinger-like equation

$$-\frac{d^2\eta}{dx^2} + V_{sch}(x)\eta = \omega^2\eta \tag{5}$$

with

$$V_{sch}(x) = V_{\phi\phi}(\phi(x)). \tag{6}$$

As one knows [11], this equation is factorizable, since it can be written as

$$S_\pm^\dagger S_\pm \eta = \omega^2\eta \tag{7}$$

with

$$S_\pm = -\frac{d}{dx} \pm W_{\phi\phi}, \tag{8}$$

which forbids the existence of tachyonic modes. Zero-mode exist, corresponding to a translational degree of freedom of the kink/antikink, and is given by

$$\eta_0 = C W_\phi, \tag{9}$$

with C a normalization constant.

Nonintegrable models like those considered here have the scattering process with a very rich character. The archetype model is the ϕ^4, where the main aspects were studied in deep (see for instance refs. [12–15]). There one knows that large initial velocities $v > v_{crit}$ lead to a simple scattering process where the kink–antikink pair encounter and after a single contact recede from themselves. This is called a 1-bounce process. Small initial velocities lead to the formation of a bound kink–antikink state called a bion that radiates continuously until being completely annihilated. Near to the frontier region bion/1-bounce, with velocities $v \lesssim v_{crit}$ it can occur the formation of two-bounce windows that accumulate toward $v = v_{crit}$ with smaller and smaller widths. At the edges of each two-bounce window another system of three-bounce windows can be found. This substructure is verified for even higher levels of bounce windows in a fractal structure [14]. Several models of kinks can be found in the recent literature, focusing on different aspects of structure and scattering dynamics from kinks [16–24].

Stability analysis of the kink leads to a Schrödinger-like equation where the presence of zero-mode is related to the translational invariance of the kink (invariance under Lorentz boosts). Usually one interprets the presence of non-null, bound states as vibrational states. In the standard Campbell–Schonfeld–Wingate (CSW) mechanism, a resonant exchange of energy between the translational and vibrational modes is responsible for the structure of two-bounce windows [13]. As far as we know there are two exceptions to this scenario: i) despite the absence of vibrational mode in the perturbation of a kink, considering the effect of collective antikink–kink structure, it was explained the reason for the occurrence of two-bounce windows in the ϕ^6 model [16]; ii) the presence of more that one vibrational state can in some circumstances destroy the two-bounce structure [22].

In this work we are interested in studying the effect of the separation of the vacua of the potential $V(\phi)$ in the process of kink–antikink collision, focusing mainly on the appearance and structure of two-bounce windows. In the Sect. 2 we will investigate a model with unusual scattering properties that can also contribute to an understanding of the mechanism of formation of two-bounce windows. In the Sect. 3 we present the numerical analysis of the kink–antikink scattering process. Our main conclusions are reported in the Sect. 4.

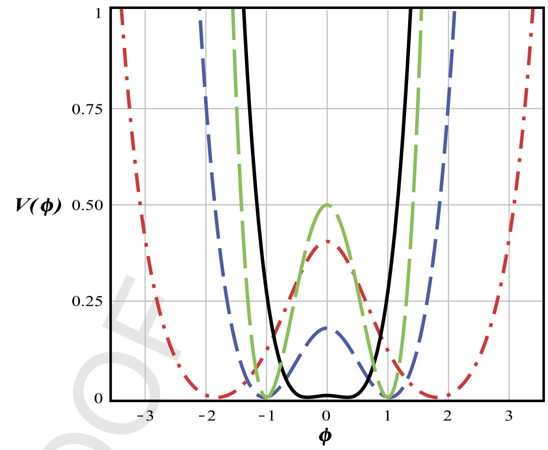

Fig. 1. Potential $V(\phi)$. The figures are for fixed $A = 0.1$ (red dash-dotted), $A = 0.4$ (blue dash), $A = 0.9$ (black line). Here it is also representing the potential for the ϕ^4 model (long-dashed green).

2. The model

An exception of the known mechanism for constructing kinks includes the so called vacuumless defects which are constructed in models with a potential that has a local maximum but no minima. One example is the model proposed by Cho and Vilenkin [25]:

$$V(\phi) = \text{sech}^2(\phi). \tag{10}$$

Potentials of this type appear also in non-perturbative effects in supersymmetric gauge theories [26,27]. The equation of motion for the scalar field has the solution

$$\phi(x) = \sinh^{-1}(\sqrt{2}x) \tag{11}$$

which has finite energy.

Static solutions for this model where further studied both in their gravitational aspects in $(3,1)$ dimensions [28] and concerning to their topological structure and trapping of fields in $(1,1)$ dimensions [29]. More recently, Dutra and Faria Jr [30] considered an extension described by the potential

$$V(\phi) = \frac{1}{2}\left(A\cosh(\phi) - \text{sech}(\phi)\right)^2. \tag{12}$$

Fig. 1 shows plots of $V(\phi)$ for several values of $0 < A < 1$, where there is the presence of two symmetric minima (due to the Z_2 symmetry) and a local maximum at $\phi = 0$. When A is reduced, the vacua are located at larger values of $|\phi|$ and the local maximum grows. The vacuumless potential from Cho and Vilenkin is recovered for $A \to 0$. Only for comparison, we plot the potential for the ϕ^4 model, showing that for the same vacua ($\phi = \pm 1$), the local maxima of the ϕ^4 potential is higher.

Static solutions for the scalar field are [30]

$$\phi_K^{(S)}(x) = \sinh^{-1}\left(\sqrt{\frac{1-A}{A}}\tanh\left(\sqrt{A(1-A)}x\right)\right), \tag{13}$$

for kink and $\phi_{\bar{K}}^{(S)}(x) = -\phi_K^{(S)}(x)$ for antikink. The vacua of the model are described by [30]

$$\phi(x \to \pm\infty) = \pm\cosh^{-1}\left(\sqrt{\frac{1}{A}}\right), \tag{14}$$

and the energy density is given by [30]

$$\rho(x) = \frac{(1-A)^2\cosh^{-4}(\sqrt{A(1-A)}x)}{1 + (1/A - 1)\tanh^2(\sqrt{A(1-A)}x)}. \tag{15}$$

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