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Unification of gauge and Yukawa couplings

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ABSTRACT

The unification of gauge and top Yukawa couplings is an attractive feature of gauge-Higgs unification models in extra-dimensions. This feature is usually considered difficult to obtain based on simple group theory analyses. We reconsider a minimal toy model including the renormalisation group running at one loop. Our results show that the gauge couplings unify asymptotically at high energies, and that this may result from the presence of an UV fixed point. The Yukawa coupling in our toy model is enhanced at low energies, showing that a genuine unification of gauge and Yukawa couplings may be achieved.

The discovery of a Higgs boson at the LHC experiments opened a new era in particle physics. Aside from being the last missing particle predicted by the Standard Model (SM), it is allowing a direct probe of the electroweak (EW) symmetry breaking sector of the SM. In particular, the fact that its mass is close to the EW scale itself, has materialised the issue of naturalness. Mass terms for scalar fields are not protected by any quantum symmetry, therefore any new physics sector that couples to it will feed into the value of the mass. In the SM, the EW scale seems to be shielded from high energy scales, like the Planck one, however, no reason for this is present in the SM itself. Another intriguing hint for new physics is the unification of gauge couplings, that occurs at high energies once one takes into account the renormalisation group evolution of the couplings. This has lead to the development of Grand Unified Theories (GUT). The fact that the mass of the top quark is close to the EW scale also suggests that the Yukawa coupling of the top may have a similar origin.

The emergence of low-scale extra-dimensions [1], mainly supported by string theory constructions, opened new avenues for model building. One of the most interesting ideas is developed in Gauge-Higgs Unification (GHU) models [2–4]. Extra-dimensional models, in fact, contain a special class of scalar fields, that arise as an additional polarisation of vector gauge fields aligned with the extra compact space. If the Higgs can be identified as such a scalar, its couplings with the fermions (the top quark in particular) are also related to the gauge coupling. In addition, mass terms for the Higgs would be forbidden by gauge invariance in the bulk of the extra-dimensions. If the gauge symmetry is suitably broken by boundary conditions, a massless scalar emerges in the spectrum, whose potential is then radiatively generated and finite [5,6].

The GHU models are rather attractive as they address, at the same time, gauge-Yukawa unification and naturalness. The main challenge is to find a gauge group, $\mathcal{G}_{\rm GHU}$, that successfully predicts the values of the SM couplings. The requirement that it contains the EW gauge symmetry of the SM, i.e. $SU(2)_L$ and the $U(1)_Y$ of hypercharge, and at the same time broken generators transforming as the Higgs doublet field, strongly limits the possible choices. Most of these possibilities, though, would seem to give incorrect predictions [7]. In this letter we show that this conclusion is modified once the energy evolution of the couplings is properly taken into account. In fact, as the extra-dimensions are to be considered as an effective theory, the unified predictions are only valid at the cut-off of the theory. However, the experimental values refer to the EW scale, and the couplings may well change due to the running via renormalisation group equations. This fact is well studied and understood in extra-dimensional GUTs [8,9]. Even though the cut-off of the effective theory may be rather low, the running in extra-dimensions is not logarithmic but follows a power law [8,10,11], thus it is much faster than in four dimensions. We will show that, taking into account the running, the tree-level predictions are strongly modified and the low energy values of the SM couplings can match the experimental values, even if starting from

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Table 1
Gauge and Yukawa couplings in the SU(3) GHU model compared to the SM values at the M_Z scale (for the Yukawa we use the top as a reference even though in this toy model the Yukawa corresponds to a down-type quark). We also include for completeness the QCD coupling.

	$SU(2)_L$ g	$U(1)_Y$ g'	Yuk. y	$SU(3)_c$ g_s
SU(3) GHU	g_{GHU}	$\sqrt{3}\, g_{GHU}$	$g_{GHU}/\sqrt{2}$	–
SM	0.66	0.35	1.0	1.2

completely different tree-level values. For the top Yukawa, the running tends to ease the tension due to the largeness of the top Yukawa at low energy compared to the gauge couplings.

1. Minimal SU(3) model with a bulk triplet

We will focus here on the simplest GHU group that allows us to embed both the EW symmetry and the Higgs: $\mathcal{G}_{GHU} = SU(3)_W$ [12]. This group, of rank 2 like the EW symmetry, contains an SU(2)×U(1) subgroup that can be identified with the gauged EW one. Furthermore, the remaining 4 broken generators correspond to a doublet of SU(2) with non-vanishing hypercharge, like the Higgs doublet in the SM. Fixing the hypercharge of the doublet fixes the relation between the SU(2) and U(1) couplings. Finally, a fermion field in the fundamental representation decomposes into a doublet and singlet of the SU(2): once the hypercharge of the Higgs candidate is fixed, the hypercharges of the doublet and singlet matches those of the left-handed quarks and the right-handed down-type ones. While we would like to describe the top quark as a bulk field, we will consider this simple model as a toy to test our idea. We want to check if the running can enhance the Yukawa coupling at low energies with respect to the unified value. Note that other SM fermions can be added as localised degrees of freedom [7,12], however, their couplings to the bulk Higgs will be suppressed, thus explaining fermion masses below the EW scale. The SU(3) predictions for the gauge and Yukawa couplings, in terms of the unified coupling g_{GHU}, are shown in Table 1 together with the SM values of the couplings at the EW scale (i.e. M_Z). For the Yukawa we consider the top Yukawa as our benchmark value because it is the largest one. It is clear that the tree-level GHU predictions are different from the SM values, however, they only apply at the cut-off of the effective theory, which may be very far from the EW scale. We show that the running will strongly modify the predictions.

We thus study the running effects in a concrete model based on a single extra-dimension compactified on an interval $S^1/\mathbb{Z}_2$. The boundary conditions at the two end points of the interval, $x_5 = 0$ and $x_5 = \pi R$ (where R is the radius of the extra-dimension), are such that the GHU group is broken to the EW one. The spectrum will thus contain massless gauge bosons plus a massless scalar associated to the broken generators. Furthermore, the bulk fermion transforming as the fundamental of $SU(3)_W$ is assigned boundary conditions such that only two massless fermions appear and we identify them with the third generation quark doublet and down-type singlet (the missing SM fermions are assumed to be localised). At low energy, therefore, the field content matches that of the SM. The running of the couplings will be affected by the presence of the Kaluza–Klein (KK) states once the mass thresholds are met, starting at $m_{KK} = 1/R$.

In Fig. 1 we show the running of the couplings as a function of the energy scale μ, normalised to the unified values as in Table 1:

$$\{g_1, g_2, g_3, g_y\} = \left\{ \frac{g'}{\sqrt{3}}, g, g_s, \sqrt{2}\, y \right\} . \qquad (1)$$

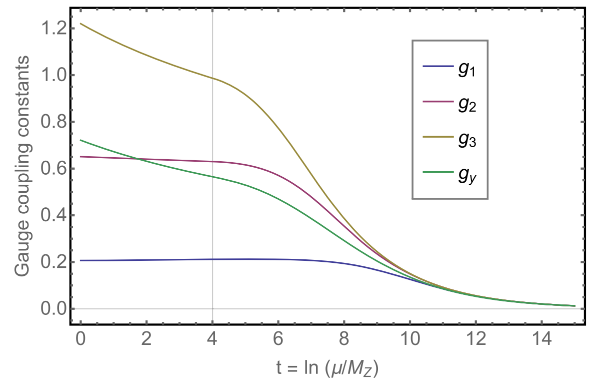

Fig. 1. Running of the normalised gauge and Yukawa couplings for the SU(3) GHU model, for $1/R = 5$ TeV. The first KK mode enters at $t_{KK} \sim 4.0$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The normalisations simply follow from the group theory structure of the $SU(3)_W$ matrices, while the QCD coupling is, in principle, unrelated. The couplings follow SM evolutions up to the scale where the first KK resonances appear, i.e.

$$t_{KK} = \ln \frac{1}{M_Z R} . \qquad (2)$$

From there on the running is modified by the extra-dimensions, and it features the expected linear behaviour. The figure clearly shows that the gauge couplings asymptotically tend to the same value.[1] This is more evident from the plot in Fig. 2, where we show, as a function of the energy, a naive estimate of the 5-dimensional loop factor, obtained by using naive dimensional analysis (NDA) [14,15]:

$$\alpha_i^{NDA}(\mu) \sim \frac{g_i^2(\mu)}{8\pi} \mu R . \qquad (3)$$

While all the couplings run asymptotically to zero, their ratio clearly tends to 1. Thus it looks as if the unified value of the gauge couplings is an UV attractor of the one loop running. It may seem surprising that the strong coupling also falls very close. However, the GHU model contains two SU(3) gauge structures, one associated to QCD and the other to the EW gauge sector, and the bulk fermion is a bi-fundamental. This allows the existence of a $\mathbb{Z}_2$ symmetry between the two sectors at high energy that implies equal couplings. Note, finally, that the NDA loop factor, which can be thought of as a 5D 't Hooft coupling (as μR counts the number of KK tiers below energy μ), can be used as a marker of the energy where the calculability of the extra-dimensional theory is lost. The fact that the values stay small seems to suggest that the theory under study may have a more extended validity than previously thought.

The initial value of the Yukawa coupling, corresponding to $y(m_Z) = 0.51$, is tuned to achieve unification in the UV. This value depends only mildly on the scale of the extra-dimension $1/R$. It should be noted that the running of the Yukawa coupling does not follow the gauge ones at high energy, due to the fact that the compactification of the extra-dimension clearly singles out the scalar component of the bulk gauge field. However, in the UV, the running needs to be replaced by the running of the 5D gauge coupling. Our results show that the value of the Yukawa coupling at low energy is larger than the values at unification, $y = g_2/\sqrt{2}$, however

[1] This behaviour for the gauge coupling evolution matches previous results, see for instance [13].