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Effect of collisions on neutrino flavor inhomogeneity in a dense neutrino gas

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ABSTRACT

We investigate the stability, with respect to spatial inhomogeneity, of a two-dimensional dense neutrino gas. The system exhibits growth of seed inhomogeneity due to nonlinear coherent neutrino self-interactions. In the absence of incoherent collisional effects, we observe a dependence of this instability growth rate on the neutrino mass spectrum: the normal neutrino mass hierarchy exhibits spatial instability over a larger range of neutrino number density compared to that of the inverted case. We further consider the effect of elastic incoherent collisions of the neutrinos with a static background of heavy, nucleon-like scatterers. At small scales, the growth of flavor instability can be suppressed by collisions. At large length scales we find, perhaps surprisingly, that for inverted neutrino mass hierarchy incoherent collisions fail to suppress flavor instabilities, independent of the coupling strength.

1. Introduction

Several recent studies, concerned primarily with anisotropic astrophysical environments, have demonstrated that the nonlinear neutrino self-coupling results in spatial instabilities [1–4]. These works indicate the importance of taking into account deviations from spherical symmetry in describing the evolution of, for example, the supernova neutrino flavor field. Previous studies of the neutrino evolution [5–9] in the early universe generally assume homogeneity and isotropy obtains at all times from the epochs of weak equilibrium through neutrino decoupling and Big Bang nucleosynthesis (BBN). The objective of the present study is to address the validity of this assumption in an exploratory calculation. To this end, in this Letter we study the stability of a two-dimensional, two-flavor dense neutrino gas with respect to the growth of seed spatial inhomogeneity driven by neutrino self-coupling. In this model, we also investigate the impact of incoherent elastic collisions on the stability properties. We find, perhaps surprisingly, that elastic scattering-angle dependent collisions are not always effective at damping flavor instabilities.

In extreme environments with large neutrino number densities, such as the interior of a core-collapse supernovae (CCSN) and in the early universe, the nonlinearity associated with neutrino self-interactions lead to an array of interesting effects beyond the "standard" MSW [10,11]. These effects, which have their origin in the non-linear character of the neutrino evolution equations, include collective oscillations in an idealized model of supernova [12–22], matter–neutrino resonant behavior [23–25], spontaneous breaking of the axial symmetry [26,27], and emergence of spatial inhomogeneity [1,2], to cite a few examples.

In this context, the early universe is particularly interesting to study since non-equilibrium and neutrino flavor oscillation effects may be important for precision cosmological probes, such as BBN. In the early universe neutrino oscillations can be induced by flavor and/or lepton asymmetries.

Studies of neutrino weak decoupling in a Boltzmann equation approach [5,9,28] suggest that neutrino spectra deviate from equilibrium Fermi–Dirac distributions at the percent level and display flavor dependence (ν_e versus ν_μ/ν_τ) at the level of a few percent [5,9]. The flavor asymmetry triggers flavor oscillations and a full analysis of this problem is still lacking. Significant progress, however, has been made in Refs. [6,7,29].

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Additionally, and complementary to the question of neutrino flavor asymmetry, neutrino oscillations play a significant role in the presence of a lepton number asymmetry, namely a difference in the spectra for neutrinos and antineutrinos (e.g. ν_e and $\bar{\nu}_e$). An electron lepton asymmetry large enough for collective neutrino oscillations to be important in the early universe can be generated in several well motivated models [30–38]. Moreover, a recent study [39], examining a range of lepton number flavor asymmetries over six orders of magnitude, identified a variety of qualitatively distinct regimes and non-trivial behavior at very small lepton asymmetry.

The departure point for this study, in light of the complex and sundry array of behavior driven by the nonlinear neutrino self-interaction, is the question of spatial inhomogeneity. We ask the question of whether the assumption of spatial homogeneity and isotropy, unexamined in previous works on the flavor field evolution in the early universe [6,7], is sustained in the presence of the nonlinear neutrino self-interaction. Our finding is that, for a two-dimensional, two-flavor exploratory model of a neutrino gas, the system is indeed unstable with respect to spatial inhomogeneity. Further, we ask the question of whether incoherent collisions, with a static background field of elastic scatterers, can serve to drive the neutrino field back to homogeneity. We investigate inhomogeneity for both normal and inverted neutrino mass hierarchies. We find that for the inverted neutrino spectrum such collisions (irrespective of the coupling strength) are incapable of stabilizing the system for a significant range of length scales.

Instability does not imply, of course, the persistence of inhomogeneity for all times. Eventually, incoherent collisions may in fact drive the system toward homogeneity. In this case, the associated transient inhomogeneity is associated with entropy generation. And, if sufficiently large, this added entropy may affect the evolution of the light element nucleosynthesis in early universe. This is a question that can only be addressed with detailed numerical simulations; such a code is currently under development [9,40,41].

Our consideration of a simplified model of the early universe is driven mainly by pragmatic concerns. The rich treasure of effects that are closely related to nonlinearity in the collective neutrino oscillations makes prediction difficult for realistic physical systems. Any such prediction involving collective neutrino oscillations in, for example, the interior of the proto-neutron star formed in CCSN requires the solution of the neutrino-flavor density matrix in a large dimensional space,[1] which is beyond the capability of even modern large-scale, multiprocessor computational platforms. However, it is possible to solve these equations for simplified models that assume deviations from highly symmetrical spatial geometries may be neglected thereby reducing the dimensionality of the solution space. It is hoped that, with this approach, insight may be gained into the development of methods that will allow the solution of the full problem; or, at least, to obtain results of phenomenological relevance.

This Letter is organized as follows. In Sec. 2, we detail the model of neutrino oscillations employed in this exploratory study of neutrino flavor stability in the early universe. Section 3 discusses the neutrino–nucleus model of the elastic, angle-dependent collision term adopted for the model. In Sec. 4, we perform linear stability analysis of the model and present results in Sec. 5. Finally, in Sec. 6, we explore the implications and limitations of the present study in the context of more realistic models. In particular, we comment on effects that inelastic contributions to the collisions may have on collective neutrino oscillations.

2. Model setup

The simplification of the spacetime and four-momentum dependence of the equations of motion of a dense neutrino gas in conditions similar to that of the early universe is effected in three stages: simplification of the geometry; restriction to two neutrino flavors; and approximation of the collision terms. The equations of motion that govern the evolution of the neutrino density matrix in the early universe depend on spacetime position and the neutrino three-momentum and energy. The equations are simplified by reducing the spatial dimensions of the universe from three to two. Since we are performing the linearized stability analysis at a given instant in the evolution of the system, we may further neglect the spacetime curvature, without loss of generality. Our assumption of elastic collisions with a background static array of nucleon-like scatterers allows the further assumption that neutrino energies are decoupled; energy transport is neglected. Thus we consider the neutrinos to be mono-energetic. This reduces the momentum dependence to that of a single, angular coordinate θ. The resulting equations of motion are four-dimensional – two spatial dimensions, one momentum direction and time. We work with two active neutrino flavors, instead of three.

The purpose of restricting the dimensionality of spacetime to a planar spatial surface is to limit the size of the space of Fourier modes (see Eq. (5) below) that must be considered in the stability analysis. In fact, we expect that this approximation is not too severe in the sense that, all other approximations taken equally, a treatment considering the universe as three spatial dimensions would simply result in a larger number of Fourier modes. The stability behaviors of at least some of the modes in this case would be similar to those of the two-dimensional case since the spatial mode wave number couples explicitly only to the local velocity of the neutrino field. The two-flavor approximation also reduces the size of the space in which the linearized stability analysis is performed. The purpose of restricting the collision term to have only angular dependence in elastic scatterings of the type $\nu_\alpha N \to \nu_\alpha N$, where the field N is represented by an infinitely massive, immobile object, is to allow consideration of just a single neutrino energy, which is conserved by this process. With these simplifications the universe can be viewed as a square of length L on each side; we may take periodic boundary conditions without loss of generality.

The dense neutrino gas is described in terms of the density matrix in flavor space with elements $f_{\alpha\beta}(x, p)$, where x and p are position and momentum four-vectors, formally related to the Wigner transform of the neutrino correlation function $\langle \nu_\alpha(x)\bar{\nu}_\beta(y)\rangle$ in medium [42–44]. Factorization of the total population (flux) $n(\vec{p})$ in momentum bin $\vec{p}$, a scalar quantity, from the correlation function yields the 2×2 density matrices $\rho^\theta(\vec{x}, t)$ and $\bar{\rho}^\theta(\vec{x}, t)$ for neutrinos and anti-neutrinos respectively, which describe the flavor content of the system; they satisfy $\mathrm{Tr}\rho^\theta = 1 = \mathrm{Tr}\bar{\rho}^\theta$. Here $\vec{x}$ is a two-dimensional vector specifying the position, θ is the polar angle characterizing the direction of the neutrino momentum $\vec{p}$, with respect to a given, arbitrary direction (taken as the y-axis); t is the time. The diagonal components of the density matrix represent the relative populations for the two neutrino flavors, denoted ν_e and ν_μ. The off-diagonal

[1] The general case of inhomogeneous and anisotropic environments, relevant for supernovae, corresponds to an equation of motion for the neutrino density matrix in a space of seven dimensions corresponding to three spatial, three momentum coordinates and time.

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