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Low energy Lorentz violation in polymer quantization revisited

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ABSTRACT

In previous work, it had been shown that polymer quantized scalar field theory predicts that even an inertial observer can experience spontaneous excitations. This prediction was shown to hold at low energies. However, in these papers it was assumed that the polymer scale is constant. But it is possible to relax this condition and obtain a larger class of theories where the polymer scale is a function of momentum. Does the prediction of low energy Lorentz violation hold for all of these theories? In this paper we prove that it does. We also obtain the modified rates of radiation for some of these theories.

1. Introduction

The problem of finding the correct quantum theory of gravity is one of the biggest challenges in physics today. While the solution to the puzzle remains out of reach, many promising approaches have been developed. On one hand there are 'top down' approaches like string theory and loop quantum gravity where one starts with a theory and tries to obtain experimental predictions. On the other there are 'bottom up' phenomenological approaches where one mainly tries to understand the consequence of Planck scale modifications of physics on the matter sector.

Polymer quantized scalar field theory [1] is a phenomenological model inspired by loop quantum gravity [2,3]. Here one first decomposes a free scalar field theory into uncoupled harmonic oscillators in momentum space and then quantizes each oscillator using polymer quantization [4] which introduces a polymer scale. This procedure yields a modified propagator which converges to the standard propagator in the limit of low energies.

Now to test any modified theory we try to find situations where its predictions conflict with the predictions of the standard theory, preferably at accessible energies. For polymer quantization, the prediction of Unruh Effect [5] (or lack thereof) has proven to be such a scenario where results obtained from polymer quantization differ significantly from standard results. Unruh Effect for polymer quantized fields in linearly accelerated frames have been studied in [6,7]. The case of rotating frames have been studied in [8]. But perhaps the most striking results have been established for inertial frames.

To understand this result, we should first note that polymer quantization violates Lorentz symmetry and establishes a preferred frame. It was shown in [9] that a detector moving with constant velocity with respect to this frame can detect radiation, if it is coupled to a polymer quantized field. Furthermore it was found that such detection occurs at low energies. In [10] it was established that there is a critical velocity such that a detector moving above this velocity will detect radiation. The rates of radiation were calculated in this paper and it was found that they cannot be suppressed by increasing the polymer scale.

However, a restrictive assumption had been made while polymer quantizing the scalar field theory in [1] It was assumed that the polymer scale is a constant. This need not be the case! Recall our description of polymer quantization of scalar field. First the field is decomposed into harmonic oscillators, one at each point in the space of spatial momenta. Then each harmonic oscillator is polymer quantized. As we will see in more detail later, this quantization requires the introduction of a scale, which we call the polymer scale. In [1], the polymer scale was assumed to be the same for all the oscillators. But clearly, this assumption can be relaxed. It is a natural extension of [1] to consider the polymer scale to be a function of $|\vec{k}|$. A running polymer scale is also natural from the perspective of renormalization group flow.

Making this extension, we arrive at a large class of polymeric theories, one for each possible $\lambda(|\vec{k}|)$. The only stipulation we must put on these theories is that they reproduce the standard field theory propagator at the low energy limit. We can now ask, do one or more of these theories not violate Lorentz symmetry at low

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energies. This is the question that we address in this paper. Surprisingly, we find that *none* of these theories can evade the fate of the original, all of them predict that an inertial detector will click at a certain critical velocity. We obtain a proof of why this should be so. We perform numerical experiments to find out the critical velocities for different theories. Surprisingly, we find that critical velocities turn out to have the same value for very different polymeric theories. Further investigation is necessary to understand why this should be so. Our result strengthens the existing result of low energy violation for polymer quantized theories. We also obtain the rates of radiations for some of these theories.

The paper is organized as follows. In the following section we recall polymer quantization of scalar fields and then modify it by introducing a momentum dependent polymer scale. In section 3 we test some of these theories numerically to see if they predict the clicking of an inertial detector and find that they do. In section 4 we give proof of why this must be so. Section 5 presents our numerical results of the rates of radiation in some of these theories. The final section summarizes our results.

2. Polymer quantization with variable polymer scale

In this section we briefly review polymer quantization of scalar field [1] and then extend it by introducing momentum dependence in the polymer scale. First, let's recall polymer quantization of a harmonic oscillator [4]. In polymer Hilbert space, the position operator $\hat{x}$ and translation operator $\hat{U}(\lambda)$ are considered to be basic operators. Since the translation operator is not weakly continuous in the parameter λ, the momentum operator does not exist in polymer Hilbert space. However, one can define the momentum operator as $\hat{p}_\lambda = 1/(2i\lambda)(\hat{U}(\lambda) - \hat{U}(-\lambda))$ and one can recover usual momentum operator by taking the limit $\lambda \to 0$. In polymer Hilbert space the limit $\lambda \to 0$ does not exist and λ is considered as a fundamental scale. By choosing λ to be λ_*, the Hamiltonian of simple harmonic oscillator can be expressed as:

$$\hat{H} = \frac{1}{8m\lambda_*^2}(2 - \hat{U}(2\lambda_*) - \hat{U}(-2\lambda_*)) + \frac{m\omega^2\hat{x}^2}{2} . \quad (1)$$

Note that it is at this step that the polymer scale enters the theory. This modifies the Schrodinger equation to:

$$\frac{1}{8m\lambda_*^2}(2 - 2\cos(2\lambda_* p))\psi - \frac{m\omega^2}{2}\frac{\partial^2\psi}{\partial p^2} = E\psi . \quad (2)$$

This can be mapped to a Mathieu equation through the following redefinitions:

$$u = \lambda_* p + \pi/2 , \quad \alpha = 2E/g\omega - 1/2g^2 , \quad g = m\omega\lambda_*^2 . \quad (3)$$

With these redefinitions the above equation takes the standard form of the Mahtieu equation:

$$\psi''(u) + (\alpha - \frac{1}{2}g^{-2}\cos(2u))\psi(u) = 0 . \quad (4)$$

This equation admits periodic solutions for certain values of α:

$$\psi_{2n}(u) = \pi^{-1/2} ce_n(1/4g^2, u), \qquad \alpha = A_n(1/4g^2) , \quad (5)$$

$$\psi_{2n+1}(u) = \pi^{-1/2} se_{n+1}(1/4g^2, u), \qquad \alpha = B_n(1/4g^2) , \quad (6)$$

where $ce_n, se_n (n = 0, 1 \ldots)$ are respectively the elliptic cosine and sine functions and A_n, B_n are the Matheiu characteristic value functions. The energy eigenvalues of the polymer harmonic oscillator are given by:

$$\frac{E_{2n}}{\omega} = \frac{2g^2 A_n(1/4g^2) + 1}{4g} , \quad (7)$$

$$\frac{E_{2n+1}}{\omega} = \frac{2g^2 B_{n+1}(1/4g^2) + 1}{4g} . \quad (8)$$

Now let us recall polymer quantization of scalar fields. Here the starting point is the free Klein Gordon field. First one takes decomposes this field into uncoupled harmonic oscillators with Hamiltonians:

$$H_{|\vec{k}|} = \frac{\pi^2_{|\vec{k}|}}{2} + \frac{|\vec{k}|^2\phi^2_{|\vec{k}|}}{2} . \quad (9)$$

Now each of these harmonic oscillators can be polymer quantized by introducing some polymer scale λ_*. In [1] each of these oscillators were quantized using the *same* polymer scale. This gives the polymer Wightman function:

$$\langle 0|\hat{\phi}(t, \mathbf{x})\hat{\phi}(t', \mathbf{x}')|0\rangle =$$
$$\sum_{n=0}^{\infty}\int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot(\mathbf{x}-\mathbf{x}')}|c_{4n+3}|^2 e^{-i\Delta E_{4n+3}(t-t')} , \quad (10)$$

where

$$\Delta E_n \equiv E_n(g) - E_0(g) , \quad (11)$$

and $c_n(g) = \langle n|\hat{\phi}_{|\vec{k}|}|0_{|\vec{k}|}\rangle$ and $g = \lambda_*^2|\vec{k}|$.

Using the asymptotic expansions for Mathieu value functions, one can obtain the propagator for low momenta ($g \ll 1$):

$$D_p = \frac{i(1 - 2\lambda_*^2|\vec{k}|)}{p^2 - \lambda_*^2|\vec{k}|^3 - i\epsilon} . \quad (12)$$

This can be seen to go to the usual limit as $g \to 0$. This completes the review of standard polymer quantization. Now we note that all the oscillators need not be polymer quantized using the same polymer scale λ_*. Oscillators corresponding to different momenta can have different polymer scales.[1] In other words, the polymer scale can be a function of momenta. In particular, since only $|\vec{k}|$ enters the oscillator Hamiltonian, the polymer scale should be taken to be a function $\mu(|\vec{k}|)$. With this modification we now have a large class of polymeric theories, one for each possible $\mu(|\vec{k}|)$. The new formula for the Wightman function and propagator will have the same form as above, with the only modification that constant λ_* will be replaced by $\mu(|\vec{k}|)$ wherever it appears. So far we have not imposed any restrictions on $\mu(|\vec{k}|)$. We will now demand that it reduces to the standard field theory propagator at the limit of low momenta. The modified polymer propagator at low energy is given by:

$$D_p^\mu = \frac{i(1 - 2|\vec{k}|\mu(|\vec{k}|)^2))}{p^2 - |\vec{k}|^3\mu(|\vec{k}|)^2) - i\epsilon} . \quad (13)$$

For this to reduce to the standard propagator we must have $\mu(|\vec{k}|)^2|\vec{k}| \to 0$ in the low momentum limit. Thus we have our only condition on $\mu(|\vec{k}|)$:

$$\mu(|\vec{k}|)^2|\vec{k}| \to 0 \text{ when } |\vec{k}| \to 0 . \quad (14)$$

[1] We note that another possible extension of polymer quantization could come from making the energy spacings field dependent. In this case the oscillators won't be governed by Mathieu equations. This would be an interesting avenue to pursue in future.

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