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Threshold energies and poles for hadron physical problems by a model-independent universal algorithm

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A B S T R A C T

In this work we show how by using a Padé type analytical continuation scheme, based on the Schlessinger point method, it is possible to find higher production thresholds in hadron physical problems. Recently, an extension of this numerical approach to the complex energy plane enabled the calculations of auto-ionization decay resonance poles in atomic and molecular systems. Here we use this so-called Resonances via Padé (RVP) method, to show its convergence beyond the singular point in hadron physical problems. In order to demonstrate the capabilities of the RVP method, two illustrations for the ability to identify singularities and branch points are given. In addition, two applications for hadron physical problems are given. In the first one, we identify the decay thresholds from a numerically calculated spectral function. In the second one, we use experimental data. First, we calculate the resonance pole of the $f_0(500)$ or σ meson using the $S0$ partial wave amplitude for $\pi\pi$ scattering in very good agreement with the literature. Second, we use data on the cross section ratio $R(s)$ for e^+e^- collisions and discuss the prediction of decay thresholds which proves to be difficult if the data is noisy.

1. Introduction and motivation

The determination of resonance poles, uniquely defined as poles of the S-matrix in the complex energy plane, is a long-standing problem and particularly difficult for broad resonances or if decay channels open up in the vicinity. In these cases, simple approaches like a standard Breit–Wigner parametrization fail and more involved theoretical tools like dispersive approaches are necessary, see e.g. [1] for reviews. However, these rigorous analytic methods require powerful mathematical techniques which makes them complicated to use in many cases.

In this letter we introduce a method that was originally developed for the calculation of auto-ionization resonances in quantum chemistry [2–4] to the field of hadron physics. This method is model-independent, easy to use and has a broad range of applicability. We refer to this method as the Resonances Via Padé (RVP) method. The RVP method is a Padé type analytical continuation scheme based on the Schlessinger point method [5] for calculating resonance poles and threshold energies. The key step in the application of this method is the identification of the analytical domain of the given function. Once this domain is identified, one can use a set of real data points from this domain, and by analytical continuation, calculate resonance poles and predict threshold energies.

Note, that there are different methods to calculate the coefficients in a Padé approximate. We use the RVP method based on the Schlessinger point method which is not equivalent to the other Padé approximates that are widely used in a large variety of fields in physics [6–8].

Let us first explain the common aspects between the RVP method, which is based on the Schlessinger point method, and between the Padé approximates as used for example in Ref. [6]. The input data in the two approaches are values of a function $F(\eta)$ on a real grid given by $\{\eta_i\}_{i=0,\pm1,\pm2,\ldots}$. The two approaches use the assumption that $\{\eta_i\}_{i=0,\pm1,\pm2,\ldots}$ are all located in the analytical domain of the function, to obtain a ratio of two polynomials

$$F(\eta) = \frac{P(\eta)}{Q(\eta)}. \quad (1)$$

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The main difference between the two methods is in the range of values of η for which the algebraic expansion of $F(\eta)$ is valid. When the Padé approximates as used for example in Ref. [6] are used the expression given in Eq. (1) holds only when $|\eta| < \eta_c$ where η_c denotes a singular point of $F(\eta)$ which is closest to the domain of the selected real grid points $\{\eta_i\}_{i=0,\pm1,\pm2,\ldots}$. Namely, one can approach the singular point from the "inside" of the set of the grid points but can not describe $F(\eta)$ beyond η_c.

However, when the RVP method is used one can describe $F(\eta)$ also beyond η_c [5]. Moreover, sufficiently close to η_c the expression given in Eq. (1) obtained by the RVP method shows a non-regular behavior. This "non-regular" behavior indicates very clearly the region where the singular point $F(\eta_c)$ is located. This ability is the main message of this paper. It enables us to study form factors and other observables and look for threshold energies and resonance poles. Up to our knowledge the convergence of an approximant beyond a singular point is unique to RVP method (see Ref. [5]) and has not been explored before by other Padé approximates. When given a finite set of M data points (η_i, F_i), it is in general not possible to find $F(\eta)$ exactly. We will therefore construct an approximation to $F(\eta)$ by using the Schlessinger point method [5]. The Schlessinger truncated continued fraction $C_M(\eta)$ is then given by

$$C_M(\eta) = \cfrac{F(\eta_1)}{1 + \cfrac{z_1(\eta-\eta_1)}{1 + \cfrac{z_2(\eta-\eta_2)}{\vdots\, z_{M-1}(\eta-\eta_{M-1})}}}, \tag{2}$$

where the z_i are real coefficients chosen such that

$$C_M(\eta_i) = F(\eta_i),\ i = 1, 2, \ldots, M. \tag{3}$$

Once the z_i are determined, an analytic continuation into the complex plane is performed by choosing η to be complex, i.e. $\eta = \alpha e^{i\theta}$. For further details on this method and the numerical implementation we refer to [2,4].

2. Two illustrations for the ability of the RVP method to identify singularities and branch points

Let us give a simple example where we compare the two methods. The considered function is

$$F(\eta) = \frac{1}{1-\eta}. \tag{4}$$

The input data are a set of points within the interval of $0 \le \eta < 1$. The one-pole Padé approximant as defined in Eq. 3 in Ref. [6] is given by

$$\mathcal{P}_1^N(\eta, \eta_0 = 0) = \sum_{n=0}^{N-1} \eta^n + \frac{\eta^N}{1-\eta}. \tag{5}$$

In Fig. 1a we show the results for $N = 5$. The excellent agreement with the $F(\eta)$ is expected since $\mathcal{P}_1^N(\eta, \eta_0 = 0)$ is an exact approximation to $F(\eta)$ in the whole space for any value of N. However, as can be seen from Fig. 1b, the one-pole Padé approach of Masjuan and Sanz-Cillero, fails to describe $F(\eta)$ close to the singularity region of $F(\eta)$ when the analytical derivatives in Eq. 3 of Ref. [6] are calculated numerically (around $\eta = 0$, using $dx = 0.0001$) or fitted (using 9 points between 0 to 1, with $R^2 = 0.9957$). On the other hand, using the RVP approach the numerical calculations from the same 5-point input data indicate very clearly on the singularity, and describes the correct behavior of $F(\eta)$ far away from the singularity at $\eta = 1$. This illustrative numerical example shows clearly the advantage of using the RVP numerical approach in the identification of the singularity of an unknown function.

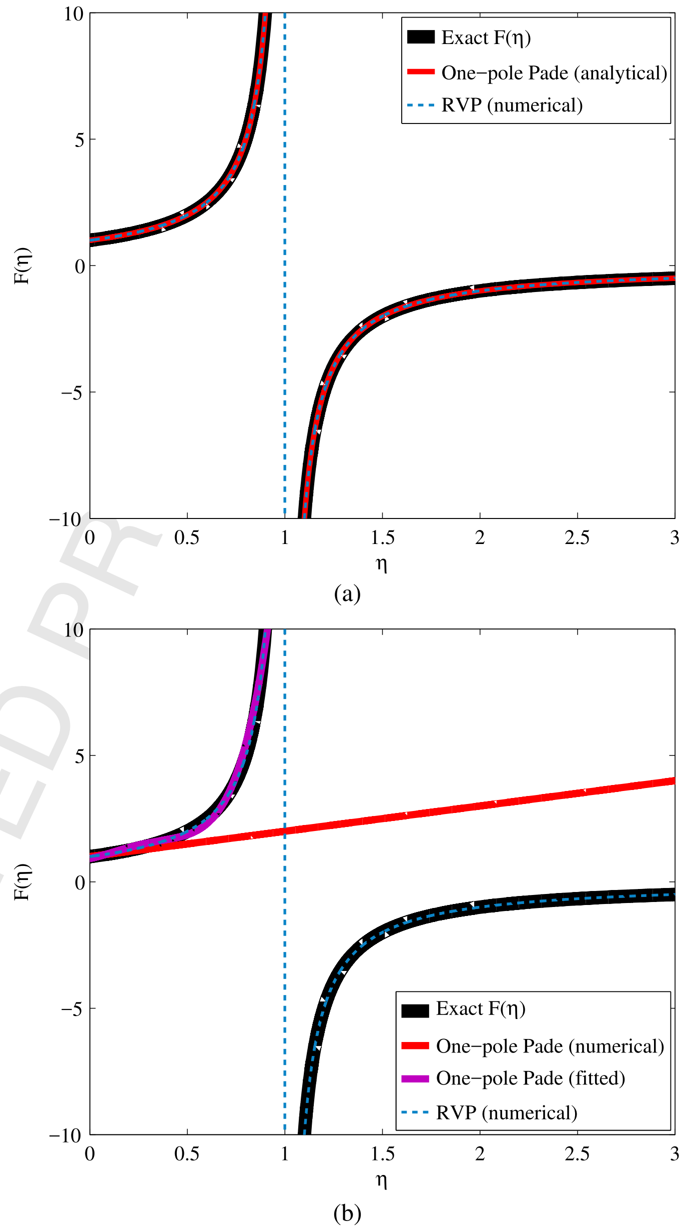

Fig. 1. (Color online.) Exact and analytically dilated plots for the function $F(\eta) = \frac{1}{1-\eta}$ from Eq. (4). (a) Analytical continuation results from the RVP approach (dashed blue line) and from the one-pole Padé approach of Masjuan and Sanz-Cillero with *analytical* derivatives (red line). Clearly, both methods accurately describe $F(\eta)$ in the whole space, and both accurately describe the singularity. (b) Analytical continuation results from the RVP approach (dashed blue line) and from the one-pole Padé approach with *numerical* derivatives (red line) and with *fitted* derivatives (purple line). Clearly, both the numerical and fitted one-pole Padé approaches fail to discover the singularity and describe $F(\eta)$ after it. Moreover, the numerical one-pole Padé approach fails to describe the function even before the singularity.

Before studying the application of the RVP approach to hadron physical problems we would like to give another illustrative example to a function of $F(\eta)$ which is non-analytical due to a branch point (BP) at $\eta_{BP} = 1$:

$$F_{BP}(\eta) = (1-\eta)^{\frac{1}{2}}. \tag{6}$$

The motivation behind this example is the fact that the BP is often associated with a bifurcation of a particle to two new parti-

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