Physics Letters B ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On the hydrodynamic attractor of Yang–Mills plasma

Michał Spaliński [a,b,*]

[a] *Physics Department, University of Białystok, 15-245 Białystok, Poland*
[b] *National Centre for Nuclear Research, 00-681 Warsaw, Poland*

ARTICLE INFO

Article history:
Received 2 November 2017
Accepted 24 November 2017
Available online xxxx
Editor: N. Lambert

Keywords:
Quark–gluon plasma
AdS/CFT

ABSTRACT

There is mounting evidence suggesting that relativistic hydrodynamics becomes relevant for the physics of quark–gluon plasma as the result of nonhydrodynamic modes decaying to an attractor apparent even when the system is far from local equilibrium. Here we determine this attractor for Bjorken flow in $\mathcal{N}=4$ supersymmetric Yang–Mills theory (SYM) using Borel summation of the gradient expansion of the expectation value of the energy momentum tensor. By comparing the result to numerical simulations of the flow based on the AdS/CFT correspondence we show that it provides an accurate and unambiguous approximation of the hydrodynamic attractor in this system. This development has important implications for the formulation of effective theories of hydrodynamics.

1. Introduction

Heavy-ion collision experiments and their phenomenological description have lead to the realization that relativistic hydrodynamics works very well rather far outside its traditionally understood domain of validity. Variants of Müller–Israel–Stewart (MIS) theory [1–3] have successfully been applied in rather extreme conditions, which could hardly be assumed to be close to local equilibrium. Furthermore, model calculations exist where it is possible to study the emergence of universal, hydrodynamic behaviour and test to what extent an effective description in terms of hydrodynamics can match microscopic results [4]. Such calculations were initially carried out in $\mathcal{N}=4$ SYM using the AdS/CFT correspondence [5–7], but similar studies have since also been performed in models of kinetic theory [8–10]. The conclusion from these investigations is that the domain of validity of a hydrodynamic description is delimited by the decay of nonhydrodynamic modes [5,6,11,12]. The outcome of this transition to hydrodynamics ("hydronization") is that the system reaches a hydrodynamic attractor [13] which governs its subsequent evolution toward equilibrium. This attractor is a special solution to which generic histories decay exponentially, and do so well before local equilibrium sets in. It incorporates all orders of the hydrodynamic gradient expansion, and at sufficiently late times coincides with the predictions of relativistic Navier–Stokes theory. The existence of an attractor in this sense is a critically important issue for hydrodynamics, because it defines its very meaning. It has conceptual as well as practical implications for the formulation of hydrodynamic theories in general as well as for their application to the physics of quark–gluon plasma.

Attractor behaviour was first identified explicitly in the differential equations of hydrodynamics [13,14]. An outstanding problem is the determination of such attractors at the microscopic level [15,4]. The first calculations of this type were described by Romatschke [15], who found approximate attractor solutions in the context of kinetic theory and $\mathcal{N}=4$ SYM by scanning for the corresponding initial conditions. The purpose of this Letter is to argue that the Borel sum of the hydrodynamic gradient expansion provides a direct way of estimating the attractor. While at late times this calculation clearly must give the correct result (which coincides with the prediction of Navier–Stokes hydrodynamics) it is not obvious *a priori* that this calculation gives an accurate estimate at earlier times. We will however show explicitly that the result of Borel summation does indeed act as an attractor for histories of Bjorken flow simulated using techniques based on the AdS/CFT correspondence. This should be viewed in the context of the idea that higher orders of the gradient expansion may be relevant for real-world physics [16–19].

An important point is that the hydrodynamic gradient expansion is the leading element of a transseries [13], and in general the higher order elements ("instanton sectors") play an important role in defining the summation properly. These transseries sectors involve integration constants which need to be fixed. However, their contributions are exponentially suppressed and it is tempting to ignore them as a first approximation. Such an approach will

* Correspondence to: National Centre for Nuclear Research, 00-681 Warsaw, Poland.
E-mail address: mspal@fuw.edu.pl.

https://doi.org/10.1016/j.physletb.2017.11.059
0370-2693/

definitely fail at sufficiently early times (before the exponential suppression sets in). However, we will see that it works fine for $\tau T > 0.3$, and this is enough to see that the result of the Borel sum acts as an attractor well before the Navier–Stokes approximation to hydrodynamics becomes accurate at $\tau T \approx 0.7$ [7].

A critical issue for Borel summation is the location of singularities of the analytic continuation of the Borel transform. These singularities reflect the spectrum of nonhydrodynamic modes – both at the microscopic level [18] and in hydrodynamics [13,14]. An important testing ground for the feasibility and robustness of Borel summation of the gradient series of $\mathcal{N} = 4$ SYM is the hydrodynamic theory proposed in [20], which we will refer to as HJSW. This theory extends Navier–Stokes hydrodynamics by adding degrees of freedom which mimic the least-damped nonhydrodynamic modes of $\mathcal{N} = 4$ SYM plasma (known from calculations of quasinormal modes of black branes [21]). This results in the same leading singularities [14] as those identified at the microscopic level in Ref. [18]. This should be contrasted with BRSSS hydrodynamics [22], which instead involves only purely decaying modes.

In the case of BRSSS theory one cannot ignore the transseries sectors even as an approximation, because the analytically-continued Borel transform of the hydrodynamic series has branch-point singularities on the real axis (reflecting the purely-decaying MIS nonhydrodynamic mode) and this leads to a complex summation ambiguity. The addition of transseries sectors (which are constrained by resurgence relations [13,14,23]) resolves this ambiguity, but requires an integration constant (the transseries parameter) to be set correctly by comparing the result of the summation to the numerical calculation of the attractor. Luckily, this issue does not arise in $\mathcal{N} = 4$ SYM, nor in HJSW hydrodynamics, because in these cases singularities of the analytic continuation of the Borel transform occur off the real axis. Thus, omitting the instanton sectors is a reasonable first approximation, which is what we focus on here.

As a way of determining the range of proper-time where the Borel sum can be expected to give an accurate estimate of the attractor we first calculate the Borel sum of the gradient expansion in the case of HJSW hydrodynamics, where it is easy to check the validity of the answer. The result is unique, unambiguous, and coincides (even at rather early times) with the attractor determined directly from the hydrodynamic equations. This sets the stage for the main theme of this Letter: the Borel summation of the gradient series of $\mathcal{N} = 4$ SYM. This is technically no more challenging than the calculation for HJSW theory, but its significance is that it provides an example of a hydrodynamic attractor obtained directly from a microscopic calculation. This result can only be fully appreciated by inspecting the behaviour of numerically simulated histories of boost-invariant expansion in $\mathcal{N} = 4$ SYM. A very important point to note is that while the attractor coincides with first order hydrodynamics at late times, it turns out to be quite distinct from it even at moderate times. This has implications of foundational nature for relativistic hydrodynamics. A fuller discussion of this result and its ramifications can be found in the concluding section.

2. Bjorken flow

Throughout this paper we work with Bjorken flow [24], which imposes powerful simplifying symmetry constraints. We use proper time – rapidity coordinates τ, Y related to Minkowski lab-frame coordinates t, z by $t = \tau \cosh Y$ and $z = \tau \sinh Y$ where z is aligned along the collision axis. A system undergoing Bjorken flow has eigenvalues of the expectation value of the energy momentum tensor

$$T^{\mu}{}_{\nu} = \mathrm{diag}(\mathcal{E}, \mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_T)^{\mu}{}_{\nu} \tag{1}$$

which are functions of the proper time τ alone. In a conformal theory, the conditions of tracelessness and conservation can be expressed as [25]

$$\mathcal{P}_L = -\mathcal{E} - \tau\,\dot{\mathcal{E}}\,, \quad \mathcal{P}_T = \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}}. \tag{2}$$

The departure of these quantities from the equilibrium pressure at the same energy density, $\mathcal{P} \equiv \mathcal{E}/3$, is a measure of how far a given state is from local equilibrium. This is conveniently captured by the pressure anisotropy

$$\mathcal{A} \equiv \frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}} \tag{3}$$

which we will study as a function not of the proper time τ, but of the dimensionless "clock variable" $w \equiv T\tau$, where T is the effective temperature (defined as the temperature of the equilibrium state with the same energy density). It is critically important to compare states of the system at different values of this dimensionless variable if we wish to see the attractor behaviour which is of central interest here.

3. The hydrodynamic attractor in hydrodynamics

Hydrodynamic theories are described by sets of nonlinear partial differential equations. The key simplification brought by the assumption of Bjorken flow is that the equations of hydrodynamics reduce to ordinary differential equations. For example, the evolution equation for the pressure anisotropy in conformal BRSSS theory reads [13,4]

$$C_{\tau_\pi}\left(1 + \frac{\mathcal{A}}{12}\right)\mathcal{A}' + \left(\frac{C_{\tau_\pi}}{3w} + \frac{C_{\lambda_1}}{8C_\eta}\right)\mathcal{A}^2 = \frac{3}{2}\left(\frac{8C_\eta}{w} - \mathcal{A}\right) \tag{4}$$

where the prime denotes a derivative with respect to w, and the dimensionless constants $C_\eta, C_{\tau_\pi}, C_{\lambda_1}$ are transport coefficients (whose values in the case of $\mathcal{N} = 4$ SYM are known, see e.g. Ref. [4]). This equation is nonlinear, but it can be solved in powers of $1/w$: this is the hydrodynamic gradient expansion whose leading term reproduces the prediction of Navier–Stokes hydrodynamics. It also possesses an attractor, which can be determined numerically by setting initial conditions appropriately [13]. It is important to observe that the attractor becomes indistinguishable from the first order truncation of the gradient series only for $w > 0.7$. For smaller values of w, the numerical solutions clearly decay to the attractor, not to the truncated gradient series.

The pressure anisotropy in HJSW theory satisfies a second order nonlinear ordinary differential equation, whose exact form can be found in Refs. [14,4], and a similar analysis leads to the numerical determination of its attractor solution (to which we shall return shortly). The point we wish to make at this juncture is that we cannot proceed in the same way in $\mathcal{N} = 4$ SYM, because there we cannot write down a closed differential equation like Eq. (4). To find the attractor in this case one has to find another way. The approach explored in this Letter is to sum the hydrodynamic gradient expansion, whose leading 240 coefficients were obtained using the AdS/CFT correspondence in Ref. [18]. In the following we discuss the properties of the series and the summation, using HJSW theory as a testing ground.