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Quark transverse spin–orbit correlations

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ABSTRACT

We extend the study of quark spin–orbit correlations in the nucleon to the case of transverse polarization. At the leading-twist level, this completes the spin structure of the quark kinetic energy–momentum tensor. In particular, we revisit the transversity decomposition of angular momentum proposed a decade ago by Burkardt and introduce a new transverse correlation, namely between quark transversity and orbital angular momentum. We also provide for the first time the Wandzura–Wilczek expression for the second Mellin moment of twist-3 transversity generalized parton distributions, along with a new sum rule. Based on lattice calculation results, we conclude that the quark transverse spin–orbit correlation is negative for both up and down flavors, just like in the longitudinal case.

1. Introduction

Understanding the nucleon spin structure is one of the key questions in hadronic physics. It opens a window on a wide range of non-perturbative effects in quantum chromodynamics (QCD) currently studied at many facilities such as Jefferson Lab, RHIC and COMPASS [1], and is a major pillar of the physics case of the future electron–ion collider (EIC) [2]. Although the proper decomposition of the nucleon spin into quark and gluon contributions constitutes one of the fundamental motivations in this field, see e.g. [3–5], the spin structure turns out to be much richer owing to spin–orbit correlations [6–8].

In a former paper [7], the quark longitudinal spin–orbit correlation was studied in detail by performing a (chiral-even) helicity decomposition of the quark energy–momentum tensor. It has, in particular, been shown that the quark longitudinal spin–orbit correlation can quantitatively be expressed in terms of parton distributions. Both current phenomenological extractions based on experimental data and lattice calculations indicate that the quark spin is, in average, opposite to the quark kinetic orbital angular momentum (OAM).

In this Letter, we discuss the quark transverse spin–orbit correlation by revisiting the (chiral-odd) transversity decomposition of the quark energy–momentum tensor considered a decade ago by Burkardt [9,10]. Mimicking the approach used by Ji to relate angular momentum contributions to generalized parton distributions (GPDs) [11], Burkardt decomposed the symmetric energy–momentum tensor and introduced accordingly the correlation between quark transversity and total angular momentum. Here we consider the more general asymmetric energy–momentum tensor leading to another transverse correlation, now between quark transversity and OAM.

The Letter is organized as follows: In section 2, we define the quark transverse spin–orbit correlation operator and express the corresponding expectation value in terms of tensor generalized form factors. In section 3 we relate these generalized form factors to moments of measurable parton distributions and derive for the first time the Wandzura–Wilczek expression for the second Mellin moment of twist-3 transversity generalized parton distributions, along with a new sum rule. In section 4, we compare the various contributions obtained on the lattice with relativistic quark model predictions, provide an estimate of the quark transverse spin–orbit correlation, and we conclude the paper with section 5.

2. Quark spin–orbit correlations

2.1. Decomposition based on polarization

It is well known that the quark field operator can be decomposed into right- and left-handed contributions

$$\psi = \psi_R + \psi_L, \qquad \psi_{R,L} = \tfrac{1}{2}(\mathbb{1} \pm \gamma_5)\psi. \tag{1}$$

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The quark number and helicity light-front operators can then respectively be seen as the *sum* and *difference*

$$\int \mathrm{d}^3x\,\overline{\psi}\gamma^+\psi = \hat{N}^q_R + \hat{N}^q_L, \tag{2}$$

$$\int \mathrm{d}^3x\,\overline{\psi}\gamma^+\gamma_5\psi = \hat{N}^q_R - \hat{N}^q_L \tag{3}$$

of the right- and left-handed densities

$$\hat{N}^q_{R,L} = \int \mathrm{d}^3x\,\overline{\psi}_{R,L}\gamma^+\psi_{R,L}, \tag{4}$$

where $a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$ for a generic four-vector a, and $\mathrm{d}^3x = \mathrm{d}x^-\,\mathrm{d}^2x_\perp$.

Alternatively, the quark field operator can be decomposed into up and down transverse polarizations [12]

$$\psi = \psi_\uparrow + \psi_\downarrow, \qquad \psi_{\uparrow,\downarrow} = \tfrac{1}{2}(\mathbb{1} \pm \gamma^j\gamma_5)\psi \tag{5}$$

with $j = 1$ or 2. While the sum of up and down densities naturally gives the quark number operator, their difference defines the so-called quark transversity

$$\int \mathrm{d}^3x\,\overline{\psi}\gamma^+\psi = \hat{N}^q_\uparrow + \hat{N}^q_\downarrow, \tag{6}$$

$$\int \mathrm{d}^3x\,\overline{\psi}i\sigma^{j+}\gamma_5\psi = \hat{N}^q_\uparrow - \hat{N}^q_\downarrow, \tag{7}$$

where

$$\hat{N}^q_{\uparrow,\downarrow} = \int \mathrm{d}^3x\,\overline{\psi}_{\uparrow,\downarrow}\gamma^+\psi_{\uparrow,\downarrow}. \tag{8}$$

The same decompositions can be performed with the quark light-front OAM operator

$$\begin{aligned}\sqrt{2}\,\epsilon^{\mu}{}_{+\alpha\beta}\int \mathrm{d}^3x\,\overline{\psi}\gamma^+x^\alpha\,\tfrac{i}{2}\overleftrightarrow{D}^\beta\psi &= \hat{L}^{q,\mu}_R + \hat{L}^{q,\mu}_L \\ &= \hat{L}^{q,\mu}_\uparrow + \hat{L}^{q,\mu}_\downarrow,\end{aligned} \tag{9}$$

where

$$\hat{L}^{q,\mu}_a = \sqrt{2}\,\epsilon^{\mu}{}_{+\alpha\beta}\int \mathrm{d}^3x\,\overline{\psi}_a\gamma^+x^\alpha\,\tfrac{i}{2}\overleftrightarrow{D}^\beta\psi_a \tag{10}$$

with the convention $\epsilon_{0123} = +1$, $a = R, L, \uparrow, \downarrow$, and $\overleftrightarrow{D}^\beta = \overrightarrow{\partial}^\beta - \overleftarrow{\partial}^\beta - 2igA^\beta$ the symmetric gauge covariant derivative. Considering instead the differences of densities leads us to longitudinal and transverse spin–orbit correlations ($\epsilon^{12}_T = -\epsilon^{21}_T = +1$ and $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$)

$$\hat{C}^q_z \equiv \epsilon^{lk}_T\int \mathrm{d}^3x\,\overline{\psi}\gamma^+\gamma_5\,x^l\,\tfrac{i}{2}\overleftrightarrow{D}^k\psi = \hat{L}^{q,+}_R - \hat{L}^{q,+}_L, \tag{11}$$

$$\hat{C}^q_j \equiv \sqrt{2}\,\epsilon^{jl}_T\int \mathrm{d}^3x\,\overline{\psi}i\sigma^{j+}\gamma_5\,x^{[-}\,\tfrac{i}{2}\overleftrightarrow{D}^{l]}\psi = \hat{L}^{q,j}_\uparrow - \hat{L}^{q,j}_\downarrow \tag{12}$$

without summation over j in (12). These are the diagonal components of a 3×3 matrix whose entries are the directions of quark polarization and OAM.

The longitudinal spin–orbit correlation (11) has been studied in [7]. In this Letter, we focus on the transverse spin–orbit correlation (12) which can conveniently be rewritten as (once again without summation over j)

$$\hat{C}^q_j = \sqrt{2}\,\epsilon^{jl}_T\int \mathrm{d}^3x\left[x^-\hat{T}^{j+l}_{q5} - x^l\hat{T}^{j+-}_{q5}\right] \tag{13}$$

with $\hat{T}^{\lambda\mu\nu}_{q5}$ the quark energy–momentum tensor where γ^μ has been replaced by $i\sigma^{\lambda\mu}\gamma_5$

$$\hat{T}^{\lambda\mu\nu}_{q5}(x) = \overline{\psi}(x)i\sigma^{\lambda\mu}\gamma_5\,\tfrac{i}{2}\overleftrightarrow{D}^\nu\psi(x). \tag{14}$$

We added the index 5 to indicate the presence of the matrix γ_5 and to distinguish it from $\hat{T}^{\lambda\mu\nu}_q = \overline{\psi}i\sigma^{\lambda\mu}\,\frac{i}{2}\overleftrightarrow{D}^\nu\psi$. These two operators are equivalent owing to the identity $i\sigma^{\mu\nu}\gamma_5 = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$.

2.2. Parametrization

We find that the non-forward matrix elements of $\hat{T}^{\lambda\mu\nu}_{q5}$ can be parametrized in terms of seven generalized form factors (GFFs)

$$\langle p',\boldsymbol{s}'|\hat{T}^{\lambda\mu\nu}_{q5}(0)|p,\boldsymbol{s}\rangle = \overline{u}(p',\boldsymbol{s}')\Gamma^{\lambda\mu\nu}_{q5}u(p,\boldsymbol{s}) \tag{15}$$

with

$$\begin{aligned}\Gamma^{\lambda\mu\nu}_{q5} = {}& \frac{P^\nu P^{[\lambda}\Delta^{\mu]}\gamma_5}{2M^2}A^q_T(t) + \frac{g^{\nu[\lambda}\Delta^{\mu]}\gamma_5}{2}\tilde{A}^q_T(t) \\ &+ \frac{P^\nu P^{[\lambda}\gamma^{\mu]}\gamma_5}{M}B^q_T(t) + M\,g^{\nu[\lambda}\gamma^{\mu]}\gamma_5\,\tilde{B}^q_T(t) \\ &+ \frac{\Delta^\nu\Delta^{[\lambda}\gamma^{\mu]}\gamma_5}{4M}C^q_T(t) + P^\nu i\sigma^{\lambda\mu}\gamma_5\,D^q_T(t) \\ &+ \frac{P^{[\lambda}i\sigma^{\mu\nu]}\gamma_5}{2}\tilde{D}^q_T(t),\end{aligned} \tag{16}$$

where $\boldsymbol{s}$ and $\boldsymbol{s}'$ are the initial and final rest-frame polarization unit vectors, M is the nucleon mass, $P = \frac{p'+p}{2}$ is the average four-momentum, and $t = \Delta^2$ is the square of the four-momentum transfer $\Delta = p' - p$. Note that the last term is totally antisymmetric over all three Lorentz indices, so that $\frac{P^{[\lambda}i\sigma^{\mu\nu]}\gamma_5}{2} = P^\nu i\sigma^{\lambda\mu}\gamma_5 + P^{[\lambda}i\sigma^{\mu]\nu}\gamma_5$. To recover the twist-2 parametrization of Hägler and Diehl [13,14], one has to symmetrize over the pair of indices $\{\mu\nu\}$, antisymmetrize over the pair of indices $[\lambda\mu]$ and remove all the traces [15]. As a result, the tilde GFFs become redundant

$$3\tilde{A}^q_T(t) \overset{\text{tw-2}}{=} (\tau-1)\,A^q_T(t) + C^q_T(t) - D^q_T(t), \tag{17}$$

$$3\tilde{B}^q_T(t) \overset{\text{tw-2}}{=} (\tau-1)\,B^q_T(t) - \tau C^q_T(t) + D^q_T(t), \tag{18}$$

$$3\tilde{D}^q_T(t) \overset{\text{tw-2}}{=} -D^q_T(t), \tag{19}$$

where $\tau = \frac{t}{4M^2}$. This means that only four GFFs survive at leading twist in agreement with the results of [13,14]. More precisely, we find that the two parametrizations at leading twist are related as follows

$$A^q_T(t) + B^q_T(t) = B_{T20}(t), \tag{20}$$

$$B^q_T(t) = 2\tilde{A}_{T20}(t) + B_{T20}(t), \tag{21}$$

$$C^q_T(t) = 2\tilde{B}_{T21}(t), \tag{22}$$

$$D^q_T(t) - B^q_T(t) = A_{T20}(t) - 2\tau\tilde{A}_{T20}(t). \tag{23}$$

We are ultimately interested in the matrix elements of Eq. (13) which involves one explicit power of x. It is therefore sufficient to expand Eq. (15) up to linear order in Δ [3,16]. Using the light-front spinors (see e.g. Appendix A of [17]) with the same rest-frame polarization $\boldsymbol{s}' = \boldsymbol{s} = (\boldsymbol{s}_\perp, s_z)$, we obtain

$$\begin{aligned}&\langle p',\boldsymbol{s}|\hat{T}^{\lambda\mu\nu}_{q5}|p,\boldsymbol{s}\rangle = \\ &\quad\left[\frac{2P^\nu P^{[\lambda}S^{\mu]}}{M} + \frac{MP^\nu\,i\epsilon^{+\lambda\mu\Delta}}{P^+}\right](B^q_T - D^q_T) \\ &\quad + \left[2M\,g^{\nu[\lambda}S^{\mu]} + \frac{M\,g^{\nu[\lambda}i\epsilon^{\mu]+P\Delta}}{P^+}\right]\tilde{B}^q_T \\ &\quad - \frac{P^\nu i\epsilon^{\lambda\mu P\Delta}}{M}B^q_T - M\,i\epsilon^{\lambda\mu\nu\Delta}\,\tilde{D}^q_T + \mathcal{O}(\Delta^2)\end{aligned} \tag{24}$$

with the covariant spin vector $S^\mu = [s_zP^+, -s_zP^- + \frac{\boldsymbol{P}_\perp}{P^+}\cdot(M\boldsymbol{s}_\perp + \boldsymbol{P}_\perp s_z), M\boldsymbol{s}_\perp + \boldsymbol{P}_\perp s_z]$ satisfying $P\cdot S = 0$ and $S^2 = -M^2(1-\tau s_z^2)$.