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A solvable three-body model in finite volume

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ABSTRACT

In this work, we propose an approach to the solution of finite volume three-body problem by considering asymptotic forms and periodicity property of wave function in configuration space. The asymptotic forms of wave function define on-shell physical transition amplitudes that are related to distinct dynamics, therefore, secular equations of finite volume problem in this approach require only physical transition amplitudes. For diffractive spherical part of wave function, it is convenient to map a three-body problem into a higher dimensional two-body problem, thus, spherical part of solutions in finite volume resembles higher spatial dimensional two-body Lüscher's formula. The idea is demonstrated by an example of two light spinless particles and one heavy particle scattering in one spatial dimension.

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1. Introduction

Three-body interaction plays an important role in many aspects of nuclear, hadron and condensed matter physics. In nuclear and astrophysics for example, the precise knowledge of nucleon interaction is the key to understand nucleon structure and dynamics of nuclei, and it is also the fundamental information to explore the origin of universe. In hadron physics, three-body dynamics could be crucial in many physical processes, such as extraction of light quark mass difference from isospin violating decay of $\eta \to 3\pi$ [1–9]. Three-body effect also have attracted a lot of interest in condense matter physics, for examples, the fractional quantum Hall states [10,11] and cold polar molecules [12]. Traditionally, three-body dynamics has been studied based on many different approaches, such as Bethe–Salpeter equations [13–15], Faddeev's equation [16–19], and dispersive approach [20–27].

Unlike the traditional three-body dynamics in free space, the finite volume three-body problem is still in a developing phase, though some progresses from different approaches have been made in recent years, such as quantum field theory based diagrammatic approach and Faddeev equation based methods [28–35], and the approach by considering periodic wave function in configuration space [36]. In contrast, the finite volume two-body problem has been well-developed [37–49] based on a pioneer work by Lüscher [50], which is usually referred to Lüscher's formula. In fact, no matter what kind of boundary conditions are considered accordingly due to relativistic effect, moving frame, *etc.*, two-body Lüscher's formula is the result of periodicity properties of asymptotic forms of wave function: (1) asymptotic form of wave function defines the physical transition amplitude and its general properties, such as unitarity relation, without consideration of specific form of interaction; (2) the periodicity of wave function yields the secular equation that relates the scattering amplitudes to periodic lattice structure and produces discrete energy spectra. As will be made clear later on, this feature of finite volume dynamics is also true in multiple-body dynamics, though because extra degrees of freedom and new types of interactions are introduced, the asymptotic form of multiple-body wave function appears more complex than two-body wave function. In general, the three-body problems are quite complex even in free space, the dynamics are usually described by off-shell unphysical amplitudes that are the solutions of Faddeev-type integral equations in momentum space. Finding the solutions of momentum space off-shell amplitudes in free space is already a uneasy task. Even in 1D space, only a few problems can be solved exactly, such as, McGuire's model in [36,51]. On the other hand, the asymptotic form of wave function in configuration space is completely determined by physical transition amplitudes [52,53]. Therefore, it seems natural to seek the solutions of finite volume three-body problems by considering asymptotic forms and periodicity of wave function in configuration space. The on-shell unitarity relation of physical amplitudes can be easily implemented by normalization of wave function in this way. The wave function approach has been proven valid and effective in finite volume two-body problems [48,49], and it has also been successfully employed to a solvable 1D three-body problem in [36]. Unfortunately, McGuire's model solved in [36] yields no diffraction effect, no

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new momenta are generated, and wave function is simply given by sum of plane waves. This letter tends to show that the wave function approach is originated from general features of multiple-body wave function, such as, asymptotic behaviors and periodicity, therefore it must be valid for finite volume multiple-body problems. Moreover, from mathematical perspective, the extra degrees of freedom in particle numbers is equivalent to the two-body scattering in higher spatial dimensions with extra types of interaction. It hence may be convenient to map the finite volume multiple-body problem to higher spatial dimensions two-body problem. For a clear demonstration of wave function approach, we consider a one spatial dimensional three-body scattering of two spinless light particles and one infinite heavy particle, the interactions among three particles are given by two types: (1) the pair-wise interaction between one light and one heavy particle, denoted as V-potential; (2) a "true" three-body interaction that all particles are involved in scattering, referred as U-potential. This 1D three-body problem is then mapped into a 2D two-body scattering problem, pair-wise potential is associated to disconnected and rescattering contributions and yields plane waves type asymptotic behavior of wave function, and "true" three-body potential results in a diffractive spherical 2D scattered wave. The quantization of three-body problem in finite volume is thus derived by taking into account of both asymptotic form and periodicity of wave function in a 2D space. In order to keep a clean and simple form in presentation, we have assumed that the pair-wise interaction between two light particles is absent, thus the disconnected contribution with heavy particle as a spectator and rescattering contributions between light-light pair and heavy-light pair vanish. The consideration of these contributions will only add some extra plane waves in asymptotic form of wave function, so neglect of these contributions won't affect the method that we tend to present, the discussion with pair-wise interaction in all pairs will be given elsewhere. In addition, we will also ignore relativistic effects and work only in the center of mass frame of three-particle. As mentioned earlier, these effects will only yield a lattice with a distorted shape and make a twist on periodic boundary condition, so that ignorance of these effects won't have much impact on our presentation as well.

2. Three-body scattering in free space

Considering scattering of two light and one infinite heavy spinless particles in a 1D space, the heavy particle is labeled as third particle, two distinguishable light particles are labeled as particle-1 and -2 with equal mass m. The relative coordinates and momenta between light and heavy particles are denoted by $r_{1,2}$ and $q_{1,2}$ respectively. The center of mass frame three-particle wave function satisfies Schrödinger equation,

$$\left[\frac{\sigma^2+\nabla^2_{r_1}+\nabla^2_{r_2}}{2m}-\sum_{i=1}^{2}V(r_i)-U(\mathbf{r})\right]\psi(\mathbf{r};\mathbf{q})=0, \tag{1}$$

where $\sigma^2=2mE=q_1^2+q_2^2$ is associated to the total center of mass energy, and short hand notation $\mathbf{r}=(r_1,r_2)$ and $\mathbf{q}=(q_1,q_2)$ are adopted throughout the presentation. Potential $V(r_{1,2})$ represents a pair-wise interaction between one light and the heavy particle, and potential $U(\mathbf{r})$ stands for a "true" three-body interaction with all three particles involved in scattering. Mathematically, the 1D three-body scattering problem given by Eq. (1) is equivalent to a two-body scattering problem in a 2D space. The three-body wave function is given by sum of multiple components, each component displays a distinct asymptotic form of scattered wave. As an example, for repulsive interactions, two distinct types of scattered waves are (i) linear superposition of plane waves with no new momenta created and describe scattering contribution due to pair-wise interactions; (ii) the diffractive wave that resembles a spherical wave in 2D two-body scattering. Hence, the technique in 2D two-body scattering can be applied in 1D three-body scattering.

For an incoming wave, $e^{i\mathbf{q}\cdot\mathbf{r}}$, the solution of three-body wave function is given by Lippmann–Schwinger equation,

$$\psi(\mathbf{r};\mathbf{q})=\phi_{q_1}(r_1)\phi_{q_2}(r_2)+\int d\mathbf{r}'G_{(12)}(\mathbf{r},\mathbf{r}';\sigma^2)2mU(\mathbf{r}')\psi(\mathbf{r}';\mathbf{q}), \tag{2}$$

where $\phi_{q_i}(r_i)$ stands for the wave function of two-body scattering between i-th ($i=1,2$) and third particle, and it satisfies Schrödinger equation $\left(q_i^2+\nabla^2_{r_i}-2mV(r_i)\right)\phi_{q_i}(r_i)=0$, and Green's function $G_{(12)}$ satisfies equation

$$\left[\sigma^2+\nabla^2_{r_1}+\nabla^2_{r_2}-2mV(r_1)-2mV(r_2)\right]G_{(12)}(\mathbf{r},\mathbf{r}';\sigma^2)=(2\pi)\delta(r_1-r_1')(2\pi)\delta(r_2-r_2'). \tag{3}$$

Asymptotically, the solution for two-body wave function is [36,49], $\phi_{q_i}(r_i)\to e^{iq_ir_i}+it(\frac{|q_i|r_i}{|r_i|},q_i)e^{i|q_ir_i|}$, where the on-shell two-body scattering amplitude can be expanded in terms of parity eigenstates [36,49], $t(q_i',q_i)=\sum_{\mathcal{P}_i=\pm}Y_{\mathcal{P}_i}(r_i)t_{\mathcal{P}_i}(|q_i|)Y_{\mathcal{P}_i}(q_i)$, where $Y_+(x)=1$ and $Y_-(x)=\frac{x}{|x|}$.

The first term on the right hand side of Eq. (2) is composed of (1) an incoming wave, (2) the disconnected scattering contribution with only one of light particles involved in scattering and another acting as a spectator, and (3) on-shell three-body rescattering contribution when U-potential is completely turned off, thus the three-body interaction is realized by rescattering effect with alternate scattering of one of two light particles off third particle. Rescattering effects are generated purely by pair-wise interactions and persist even when U-potential is zero. The second term on the right hand side of Eq. (2) represents a "true" three-body interaction. Let's define a scattering T-amplitude that is associated to U-potential by

$$-\frac{T_{(12)}(\mathbf{k};\mathbf{q})}{\sigma^2-k^2}=\int d\mathbf{r}d\mathbf{r}'e^{-i\mathbf{k}\cdot\mathbf{r}}G_{(12)}(\mathbf{r},\mathbf{r}';\sigma^2)2mU(\mathbf{r}')\psi(\mathbf{r}';\mathbf{q}), \tag{4}$$

where $k^2=k_1^2+k_2^2$. Hence, we can rewrite Eq. (1) to

$$\psi(\mathbf{r};\mathbf{q})=\phi_{q_1}(r_1)\phi_{q_2}(r_2)-\int\frac{d\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{r}}\frac{T_{(12)}(\mathbf{k};\mathbf{q})}{\sigma^2-k^2}. \tag{5}$$

It can be easily show that [52,53]

$$T_{(12)}(\mathbf{k};\mathbf{q})=-\int d\mathbf{r}'\phi_{-k_1}(r_1')\phi_{-k_2}(r_2')2mU(\mathbf{r}')\psi(\mathbf{r}';\mathbf{q}). \tag{6}$$

For repulsive interactions, because of the absence of two-body bound states, $T_{(12)}$ displays no primary singularities [16,17], and describes the "true" three-to-three particles scattering process, which will be denoted by $T_{0,0}$ from now on. The asymptotic form of wave function thus is given by,

$$\psi(\mathbf{r};\mathbf{q})\to\phi_{q_1}(r_1)\phi_{q_2}(r_2)+\frac{ie^{i(\sigma r-\frac{\pi}{4})}}{2\sqrt{2\pi\sigma r}}T_{0,0}(\frac{\sigma\mathbf{r}}{r};\mathbf{q}), \tag{7}$$

where $r=\sqrt{r_1^2+r_2^2}$. The $T_{0,0}$-amplitude is constrained by unitarity relation,

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