Physics Letters B 774 (2017) 676–681

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On thermal gravitational contribution to particle production and dark matter

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ARTICLE INFO

Article history:
Received 23 August 2017
Received in revised form 20 September 2017
Accepted 18 October 2017
Available online 20 October 2017
Editor: J. Hisano

ABSTRACT

We investigate the particle production from thermal gravitational annihilation in the very early universe, which is an important contribution for particles that might not be in thermal equilibrium or/and might only have gravitational interaction, such as dark matter (DM). For particles with spin $0, 1/2$ and 1 we calculate the relevant cross sections through gravitational annihilation and give the analytic formulas with full mass-dependent terms. We find that DM with mass between TeV and 10^{16} GeV could have the relic abundance that fits the observation, with small dependence on its spin. We also discuss the effects of gravitational annihilation from inflatons. Interestingly, contributions from inflatons could be dominant and have the same power dependence on Hubble parameter of inflation as that from vacuum fluctuation. Also, fermion production from inflaton, in comparison to boson, is suppressed by its mass due to helicity selection.

1. Introduction

The accumulated firm evidence for dark matter (DM) has challenged modern particle physics for decades. From the galactic rotation curves to galaxy cluster, large scale structure (LSS) and cosmic microwave background (CMB), the existence of DM has been well-established, based only on the gravitational interaction. Numerous models for DM has also been proposed, see Refs. [1,2] for reviews. Broadly speaking, for DM as elementary particles, it either can be in thermal equilibrium with other particles and then freeze out, or was never in equilibrium but still produced gradually through various processes. The first class is usually referred as weakly-interacting massive particle (WIMP), while the second includes axion, sterile neutrino, gravitino and so on.

All the mentioned DM candidates above inevitably have interactions other than gravitation, therefore in principle could give rise to possible signatures in direct, indirect and collider searches. However, so far there is no confirmed evidence in all those searches for DM's new interaction, it is fair to ask what if DM only has gravitational interaction. Recent studies [3,4] have shown that it is viable to generate scalar DM abundantly with only gravitational annihilation, namely particles in the thermal both annihilate into DM through a graviton, the quantum of Einstein's gravity in the weak-field limit. Scenarios and phenomenologies in extended theories are also discussed, for example in Refs. [5–10].

In the view of effective field theories, microscopically gravity can be treated effectively as quantum field theory as long as the energy scale is much lower than Planck scale ($M_P = 1.12 \times 10^{19}$ GeV) [11,12]. Recently, it has also been shown that general relativity can be derived as an effective field theory of gravitational quantum field theory with spin and scaling gauge symmetries [13,14]. Since the energy scale during/after inflation has already been constrained to be $\lesssim 10^{16}$ GeV which is much lower than M_P, we would expect that the local scattering and/or annihilation through graviton can be described in effective field theory. Then these processes should in principle contribute to the cosmological evolution of all particle species, including DM. Particles with interactions much stronger than gravity would be in thermal equilibrium with other particles and short-range gravitational processes are essentially irrelevant for them. However, if DM is very weakly interacting and was never in equilibrium in the early universe, we should include the contributions from gravitational processes.

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https://doi.org/10.1016/j.physletb.2017.10.034
0370-2693/

In this paper, we investigate the viable mass range for DM with spin 0, 1/2 and 1, produced by the gravitational annihilation of particles in the thermal bath with various spins. We compute all the possible, general annihilation cross sections analytically, including all the finite mass term. We find that for the production from particles in the thermal bath the abundance of DM is tightly related with the highest temperature $T_{\max}$ after inflation, proportional to $T^3_{\max}/M^3_P$ if its mass $m_X < T_{\max}$ and $m^3_X/M^3_P \exp[-2m_X/T_{\max}]$ if $m_X > T_{\max}$. We also discuss the effects from inflation dynamics and show that, gravitational annihilation from inflatons might be the dominant channel for scalar/vector DM production (there is a suppression factor for fermionic DM due to helicity selection) and interestingly has the same power dependence on Hubble parameter as production from vacuum fluctuation.

This paper is organized as follows. In Sec. 1 we start with the standard Boltzmann equation to follow the cosmological evolution of particles and establish the convention and terminology for later discussions. Then in Sec. 3 we calculate the gravitational annihilation cross section for different initial and final states with spin 0, 1/2 and 1. Later in Sec. 4 we apply our calculated cross section to DM and investigate the viable mass range. In Sec. 5 we discuss the effects from chaotic inflation and show that inflaton's contribution can be very important. Finally, we give the summary.

2. Boltzmann equation

To be self-contained, let us start with the standard Boltzmann equation in cosmology [15] for the evolution of number density n_3 through the $2 \leftrightarrow 2$ process,[1] $p_1 + p_2 \leftrightarrow p_3 + p_4$,

$$\begin{aligned}
&\dot{n}_3 + 3Hn_3 \\
&\equiv \frac{d\left(a^3 n_3\right)}{a^3 dt} \\
&= \int \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1}\frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2}\frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3}\frac{d^3\boldsymbol{p}_4}{(2\pi)^3 2E_4} \\
&\quad \times (2\pi)^4\delta^4(p_1+p_2-p_3-p_4) \times \\
&\quad \sum_{\text{pol}}\Big[f_1 f_2(1\pm f_3)(1\pm f_4)\left|\mathcal{M}_{12\to 34}\right|^2 \\
&\quad - f_3 f_4(1\pm f_1)(1\pm f_2)\left|\mathcal{M}_{34\to 12}\right|^2\Big], \qquad (2.1)
\end{aligned}$$

where a is the scalar factor, Hubble parameter $H = \dot{a}/a$, $\boldsymbol{p}_i$ denote the spatial momenta, p_i for 4-vector, $\mathcal{M}$ is the matrix element, f_i is the distribution for particle i without internal degree of freedom, $+(-)$ sign in $\pm$ is for bosons (fermions) and $\sum_{\text{pol}}$ means the sum of all polarizations. For particles that were in thermal equilibrium, such as WIMP, we need to keep both terms in the bracket of Eq. (2.1). This is due to the cross symmetry $\mathcal{M}_{12\to 34} = \mathcal{M}_{34\to 12}$ and $f_1 f_2$ is compatible to $f_3 f_4$ for $E_i \sim m_3$ where m_3 is the mass for particle 3. In cases where $f_{3,4}$ is much smaller than 1 and/or $f_{1,2}$, we can neglect the second term and the above Boltzmann equation becomes

$$\begin{aligned}
\frac{d\left(a^3 n_3\right)}{a^3 dt} &= \int \frac{f_1 d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1}\frac{f_2 d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \\
&\quad \times \Bigg[\frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3}\frac{d^3\boldsymbol{p}_4}{(2\pi)^3 2E_4} \\
&\quad \times (2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\sum_{\text{pol}}\left|\mathcal{M}_{12\to 34}\right|^2\Bigg], \qquad (2.2)
\end{aligned}$$

[1] Following the same formalism, processes with multiple initial or final states can also be included. These contributions could also be important unless they are suppressed by additional small couplings or phase space factors.

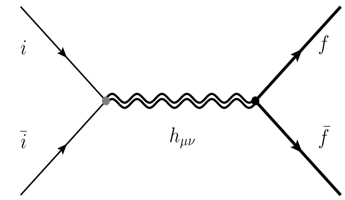

Fig. 1. Annihilation process for $i \to f$, where particles i and f can be scalars S, fermions F (spin 1/2), massive vectors V and massless vectors γ. For massive particles, we always denote the initial states'mass as m and the final states' as M. The double lines represent the graviton field, $h_{\mu\nu}$. Arrows mean the directions of momenta. Note that although i and f might have the same spin, they have to be different particles to affect the number density in Boltzmann equation.

The term in the bracket can be replaced by $4\mathcal{F}g_1 g_2 \sigma_{12\to 34}$, where g_i is the spin degree of freedom, $\sigma \equiv \sigma_{12\to 34}$ is the cross section and $\mathcal{F} = [(p_1\cdot p_2)^2 - m_1^2 m_2^2]^{1/2}$. So we have

$$\frac{d\left(a^3 n_3\right)}{a^3 dt} = \int \frac{f_1 g_1 d^3\boldsymbol{p}_1}{(2\pi)^3 E_1}\frac{f_2 g_2 d^3\boldsymbol{p}_2}{(2\pi)^3 E_2} F\sigma, \qquad (2.3)$$

Changing to the integration variables E_1, E_2 and s, we have

$$d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2 = 4\pi^2 E_1 E_2 dE_1 dE_2 ds = 2\pi^2 E_1 E_2 dE_+ dE_- ds, \qquad (2.4)$$

where $E_+ = E_1 + E_2$, $E_- = E_1 - E_2$, and $s = (p_1 + p_2)^2$. As will be shown in next section, throughout our discussion, we have $m_1 = m_2 = m$ and $m_3 = m_4 = M$ and the integration range then can be simplified to

$$\begin{aligned}
&s \ge \max(4m^2, 4M^2),\ E_1 \ge m,\ E_2 \ge m,\ E_+ \ge \sqrt{s}, \\
&|E_-| \le \sqrt{1-4m^2/s}\sqrt{E_+^2 - s}. \qquad (2.5)
\end{aligned}$$

So far, the discussions have been quite general and apply for other very weakly interacting particles as well, see Ref. [16] for a recent review. It is evident that the key part is to calculate the annihilation cross section σ. After that we can perform numerical integration or analytic computation for some special cases. If $f_{1,2}$ have quantum statistical distributions, like Fermi-Dirac or Bose–Einstein distributions $(e^{E/T} \pm 1)^{-1}$, no compact analytic formulas can be derived. However, for $E > T$, we can use approximate Maxwell–Boltzmann distribution, $e^{-E/T}$, and then integrate over E_- and E_+ to get

$$\frac{d\left(a^3 n_3\right)}{a^3 dt} = \frac{g_1^2 T}{32\pi^4}\int ds\,\sigma\,\sqrt{s}(s-4m^2)K_1\left(\frac{\sqrt{s}}{T}\right), \qquad (2.6)$$

where K_i is the modified Bessel function of the second kind with order i.

3. Annihilation cross section

In this section, we compute the annihilation cross section in the center-of-mass (CM) frame for various initial and final states in Fig. 1. Note that the initial particles are different from the final ones so that the process can change the number density and contribute to Boltzmann equation, although in a broader context for other physics problems they can be the same. Since the cross section is a Lorentz-invariant quantity, the results derived here will also be valid in other frames.

In effective field theory, the leading interactions between graviton and matter are described by