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Fine features of parametric X-ray radiation by relativistic electrons and ions

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ABSTRACT

In present work within the frame of dynamic theory for parametric X-ray radiation in two-beam approximation we have presented detailed studies on parametric radiation emitted by relativistic both electrons and ions at channeling in crystals that is highly requested at planned experiments. The analysis done has shown that the intensity of radiation at relativistic electron channeling in Si (110) with respect to the conventional parametric radiation intensity has up to 5% uncertainty, while the accuracy of approximate formulas for calculating parametric X-ray radiation maxima does not exceed 1.2%. We have demonstrated that simple expressions for the Fourier components of Si crystal susceptibility χ_0 and $\chi_{g\sigma}$ could be reduced, as well as the temperature dependence for radiation maxima in Si crystal (diffraction plane (110)) within Debye model. Moreover, for any types of channeled ions it is shown that the parametric X-ray radiation intensity is proportional to $z^{2-b(Z,z)/z}$ with the function $b(Z,z)$ depending on the screening parameter and the ion charge number $z = Z - Z_e$.

1. Introduction

Since the first observation of parametric X-ray radiation (PXR) reported in [1,2], in successful works [3–5] it was shown that the observed radiation peaks represent a new type of radiation different from diffraction of bremsstrahlung radiation. For more than 30 years PXR has been intensively studied by many groups in the world revealing particular behaviors of such radiation. For the basic theory developed and main experimental results obtained for relativistic electrons we can refer to the books [6,7], while some details in a fine structure of parametric radiation from relativistic electrons in a crystal, for example, spectral linewidth and features of orientational dependence for differential PXR yield – to the paper [8]. First studies on PXR by heavy charged particles (protons and ions) of moderate energies have been presented in [9–11], while for highly relativistic protons (Pb ions) the first observation results were published in [12,13].

As shown by many authors in the past one of the most interesting characteristics of PXR proves the feasibility of its use for the experiment diagnostic purposes. Various behaviors of this radiation are strongly related to the parameters of media, in which the radiation is formed from coherent radiation by projectiles. Moreover, the PXR intensity depends on the projectile charge that makes possible to manage the radiation characteristics. This dependence is still under investigation.

In this work we are going to present the results on investigation of some new peculiarities of parametric radiation. First of all, we have indicated the main difference between radiations at (PXRC[1]) and without channeling (PXR) of charged particle beams. Then using the QED expression of PXR angular distribution, developed in [16,17], the reasonably accurate estimates of the expressions for the PXR intensity maxima is given. To perform necessary calculations, analytical approximate dependence of PXR photon energy on Fourier components of the crystal susceptibility recently derived is reported. This expression allows within the frame of Debye model the temperature dependence of PXR spectrum to be defined. The

[1] If charged particle beams are moving in crystals at channeling conditions [14,15], they can also emit parametric X-ray radiation, PXR at channeling – PXRC, due to their interaction with various adjacent planes. Hence, PXRC is a phenomenon different from known PXR by quasi-free beams.

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latter becomes very useful for new experiments on PXR studies at present and future high current accelerator and storage ring facilities.

Finally, performing an analysis within the frame of Thomas–Fermi–Dirac (TFD) theory we have proposed an simple approximate expression for Z-dependence of the intensity of ion-induced PXR updated with respect to previous known dependence.

2. PXRC intensity versus PXR one

As previously shown theoretically [17,18] and then proved experimentally [19], PXRC emitted by electrons differs from conventional PXR by the form-factor F_{nn} and the initial population $P_n = P_n(\theta_0, k_\perp)$ of the channeling state n

$$dN_{\rm PXRC} = \frac{d^3 N_{\rm PXRC}}{d\theta_x d\theta_y dz} = dN_{\rm PXR} \sum_n P_n |F_{nn}|^2 \,, \qquad (1)$$

where $dN_{\rm PXR}$ is the standard PXR angular distribution [17],

$$dN_{\rm PXR} = \frac{\alpha\omega_B}{16\pi c \sin^2\theta_B}\left[\frac{\theta_x^2}{1+W_\pi^2} + \frac{\theta_y^2}{1+W_\sigma^2}\right], \qquad (2)$$

F_{nn} is the matrix element describing the probability of spontaneous channeled electron intraband transition within the band n accompanied by the emission of a photon in the Bragg direction, $\omega_B = c\pi/d\sin\theta_B$ is the Bragg frequency [7], $\alpha = e^2/\hbar c$ is the fine structure constant, d is the crystal interplane distance, $\theta_o = \arctan(p_\perp/p_\parallel)$ is the angle of incidence with respect to the crystallographic plane, $p_\parallel = p_z$ and $p_\perp = \hbar k_\perp = p_y$ are the longitudinal and transverse components of electron momentum.

In Eq. (2) we have used the following denominations for the polarization type $\tau = (\pi, \sigma)$ [17,19]

$$W_\tau = \frac{1}{2|\chi_g| P_\tau}\left(R - \frac{|\chi_g|^2 P_\tau^2}{R}\right),$$

$$R = \theta_x^2 + \theta_y^2 + \theta_{kin}^2,\ \theta_{kin}^2 = \gamma^{-2} + |\chi_0|,$$

$$P_\pi = \cos 2\theta_B,\ P_\sigma = 1, \qquad (3)$$

where χ_0 and χ_g are the Fourier components of crystal dielectric susceptibility, γ is the projectile relativistic factor, τ is the type of polarization for PXRC-photon (equals to either π or σ polarization), θ_x and θ_y are the angular coordinates for deviation of the wave vector of PXRC-photon from the Bragg direction.

Thus, relative difference factor

$$\Delta_{PXRC} = \frac{dN_{\rm PXRC}}{dN_{\rm PXR}} = \sum_n P_n(\theta_0, k_y)|F_{nn}|^2 \qquad (4)$$

for the angular distributions of PXR and PXRC can be calculated only taking into account the form-factor F_{nn} and initial population $P_n(\theta_0, k_y)$.

Fig. 1 represents the dependence of Δ_{PXRC} on longitudinal electron energy $E_\parallel = \gamma mc^2$ at channeling in Si (110). Sharp jumps in the graph of difference factor Δ_{PXRC} takes place due to zone structure of transverse energy levels $E_\perp$ for channeled electrons.

The large uncertainty range of the values Δ_{PXRC} for these points is a consequence of the energy band structure for the transverse motion. The uncertainly is the difference between PXRC and PXR intensities that could be measured near the "quantum jump", which takes place at the energies of the electron beam when a new odd level of the transverse channeling state appears. The maximum values of relative difference Δ_{PXRC} correspond to the

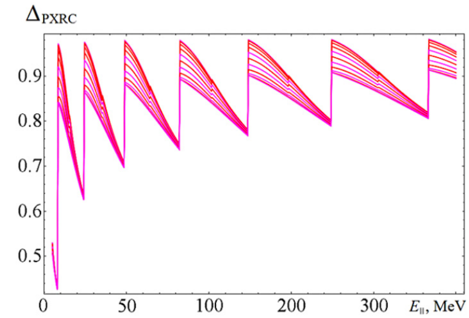

Fig. 1. Difference factor for PXRC and PXR – Δ_{PXRC} – as a function of $E_\parallel$ for (110) Si channeling of electrons (diffraction on planes (111)). The large uncertainty range of values of Δ_{PXRC} is a consequence of energy levels band structure for the transverse motion.

"lower" (in magnitude of the electron wave vector) edge of the energy band. Such unusual plot of relative difference Δ_{PXRC} reveals the PXRC intensity measurements, in contrast to PXR, to be necessarily characterized by an uncertainty for electrons channeled along (110) planes, i.e., for Si and C – up to 5%, while for LiF – up to 12% [18].

In recent paper [20], theoretical calculations for PXR intensities taking into account multiple scattering by means of the convolution technique were presented. The results obtained proves the fact that accounting for the process of multiple scattering does not practically change the magnitude of PXR maximum (Fig. 1 shows the PXR maxima). Moreover, it was shown that multiple scattering does not affect the magnitudes of transverse beam energies, at which new quantum levels of channeled motion appear.

3. Approximation for PXR maxima calculation

Analysis of Eqs. (1)–(2) proves that the maxima of PXR and PXRC could be observed at the same angles θ_{xmax} and θ_{ymax}. To get a precise expression for that is very routine, while a simple but sufficiently accurate approximation for extreme angles θ_{xmax} and θ_{ymax} can be obtained. According to Eq. (2) PXR reaches its maxima at the extreme points $f_x = \theta_{xmax}^2/(1+W_\pi^2)$ for θ_{ymax} and $f_y = \theta_{ymax}^2/(1+W_\sigma^2)$ for θ_{ymax}, where these points have been defined as the functions of θ_{kin} and χ_g. Due to the smallness of one of the parameters, $\chi_g \ll 1$, these functions could be reduced via their expansion in series up to quadratic terms $\chi_{g\tau}^2$ to rather simple equations

$$f_x \simeq \frac{4\theta_x^2\chi_{g\pi}^2}{(\theta_{kin}^2+\theta_x^2)^2}\cos^2 2\theta_B,\quad f_y \simeq \frac{4\theta_y^2\chi_{g\sigma}^2}{(\theta_{kin}^2+\theta_y^2)^2}\,, \qquad (5)$$

with the maxima at the points (corresponding to the PXR intensity maxima)

$$\widetilde{\theta}_{xmax}^{\pm} = \widetilde{\theta}_{ymax}^{\pm} = \pm\theta_{kin} \qquad (6)$$

Using these approximate expressions for the angular coordinates of PXR intensity maxima $\widetilde{\theta}_{xmax} = \widetilde{\theta}_{ymax} = \theta_{kin}$ we obtain reasonably accurate estimates for the expressions of PXR intensity maxima

$$dN_{\rm PXR}^{xmax} = \frac{\alpha\,\omega_B}{\pi c}\,\frac{\theta_{kin}^6\chi_{g\pi}^2\cos^2 2\theta_B}{(4\theta_{kin}^4+\chi_{g\pi}^2\cos^2 2\theta_B)^2}\,,$$

$$dN_{\rm PXR}^{ymax} = \frac{\alpha\,\omega_B}{\pi c}\,\frac{\theta_{kin}^6\chi_{g\sigma}^2}{(4\theta_{kin}^4+\chi_{g\sigma}^2)^2}\,, \qquad (7)$$

for the Bragg frequency of PXR with normalized speed $\beta = v/c$, where v is the electron velocity,