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Holographic superconductivity from higher derivative theory

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ABSTRACT

We construct a 6 derivative holographic superconductor model in the 4-dimensional bulk spacetimes, in which the normal state describes a quantum critical (QC) phase. The phase diagram $(\gamma_1, \hat{T}_c)$ and the condensation as the function of temperature are worked out numerically. We observe that with the decrease of the coupling parameter γ_1, the critical temperature $\hat{T}_c$ decreases and the formation of charged scalar hair becomes harder. We also calculate the optical conductivity. An appealing characteristic is a wider extension of the superconducting energy gap, comparing with that of 4 derivative theory. It is expected that this phenomena can be observed in the real materials of high temperature superconductor. Also the Homes' law in our present models with 4 and 6 derivative corrections is explored. We find that in certain range of parameters γ and γ_1, the experimentally measured value of the universal constant C in Homes' law can be obtained.

1. Introduction

The AdS/CFT correspondence [1–4] provides a powerful tool to study the quantum critical (QC) dynamics, which are described by CFT and are strongly coupled systems without quasi-particles descriptions [5]. An interesting and important holographic QC dynamical system is that a probe Maxwell field coupled to the Weyl tensor $C_{\mu\nu\rho\sigma}$ in the Schwarzschild–AdS (SS–AdS) black brane background, which is a 4 derivative theory and has been fully studied in [6–15]. This system has zero charge density and can be understood as particle-hole symmetry. Of particular interest is the nontrivial optical conductivity due to the introduction of Weyl tensor, which is similar to the one in the superfluid-insulator quantum critical point (QCP) [6–8]. Further, higher derivative (HD) terms are introduced and the optical conductivity is studied in [15]. They find that an arbitrarily sharp Drude-like peak can be observed at low frequency in the optical conductivity and the bounds of conductivity found in [6,9] can be violated such that we have a zero DC conductivity at specific parameter. Especially, its behavior resembles quite closely that of the $O(N)$ $NL\sigma M$ in the large-N limit [16]. Therefore, the HD terms in SS–AdS black brane background provide possible route and alternative way to study the QC dynamics described by certain CFTs.

In this paper, we intend to study the holographic superconductor model in this holographic framework including HD terms. The holographic superconductor model [17–19] is an excellent example of the application of AdS/CFT in condensed matter theory (CMT), which provides valuable lessons to access the high temperature superconductor in CMT. In the original version of the holographic superconductor model [17–19], the superconducting energy gap is $\omega_g/T_c \approx 8$. This value is more than twice the one, which is 3.5, in the weakly coupled BCS theory, but roughly approximates that measured in high temperature superconductor materials [20]. Furthermore, by introducing 4 derivative term based on Weyl tensor, the holographic superconductor in the boundary theory dual to 5 dimensional AdS black brane is firstly constructed in [21]. This model exhibits an appealing characteristic of the extension of superconducting energy gap approximately varying from 6 to 10 [21]. Next, lots of Weyl holographic superconductor models, including p wave and different backgrounds, are constructed in [22–32]. In particular, the extension of superconducting energy gap is also observed in [22,31].[1] Here, we extend the analysis to include the 6 derivative term in the 4 dimensional bulk spacetime, in which the

[1] In [23–30], they study the s or p wave superconducting condensation from 4 derivative term in 5 dimensional AdS geometry. But the computation of conductivity is absent. In [31], the Weyl holographic superconductor in the 4 dimensional Lifshitz black brane is explored and the extension of the superconducting energy gap is also observed. In addition, we would also like to point out that, the running of superconducting energy gap is also observed in other holographic superconductor model from higher derivative gravity, for example, the Gauss–Bonnet gravity [33,34]

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normal state describes a QC phase and the electromagnetic (EM) self-duality loses.

We also study the Homes' law over our model. Homes' law is an empirical law universally discovered in experiments of superconductors, which states that the product of the DC conductivity $\sigma_{DC}(T_c)$ and the critical temperature T_c has a linear relation to the superfluid density $\rho_S(T=0)$ at zero temperature. Holographic investigation of Homes' law can be found in [32,37–39]. Our results show that the constant of the Homes' law can be observed in certain range of parameter γ and γ_1 in our model, which can be extended by adding additional structures to study the universal realization of the Homes' law in holography.

2. Holographic framework

We shall construct a charged scalar hair black brane solution based SS–AdS black brane

$$ds^2 = \frac{L^2}{u^2}\Big(-f(u)dt^2 + dx^2 + dy^2\Big) + \frac{L^2}{u^2 f(u)}du^2\,,$$
$$f(u) = (1-u)p(u)\,, \qquad p(u) = u^2 + u + 1\,. \tag{1}$$

$u = 0$ is the asymptotically AdS boundary while the horizon locates at $u = 1$. The Hawking temperature of this system is $T = 3/4\pi L^2$. And then, we introduce the actions for gauge field A and charged complex scalar field Ψ

$$S_A = \int d^4x\sqrt{-g}\Big(-\frac{L^2}{8g_F^2}F_{\mu\nu}X^{\mu\nu\rho\sigma}F_{\rho\sigma}\Big)\,, \tag{2}$$

$$S_\Psi = \int d^4x\sqrt{-g}\Big(-|D_\mu\Psi|^2 - m^2|\Psi|^2\Big)\,. \tag{3}$$

In the action S_A, $F = dA$ is the curvature of gauge field A and the tensor X is an infinite family of HD terms as [15]

$$\begin{aligned} X_{\mu\nu}{}^{\rho\sigma} = {}& I_{\mu\nu}{}^{\rho\sigma} - 8\gamma_{1,1}L^2C_{\mu\nu}{}^{\rho\sigma} - 4L^4\gamma_{2,1}C^2 I_{\mu\nu}{}^{\rho\sigma} \\ & - 8L^4\gamma_{2,2}C_{\mu\nu}{}^{\alpha\beta}C_{\alpha\beta}{}^{\rho\sigma} - 4L^6\gamma_{3,1}C^3 I_{\mu\nu}{}^{\rho\sigma} - 8L^6\gamma_{3,2}C^2C_{\mu\nu}{}^{\rho\sigma} \\ & - 8L^6\gamma_{3,3}C_{\mu\nu}{}^{\alpha_1\beta_1}C_{\alpha_1\beta_1}{}^{\alpha_2\beta_2}C_{\alpha_2\beta_2}{}^{\rho\sigma} + \ldots\,, \end{aligned} \tag{4}$$

where $I_{\mu\nu}{}^{\rho\sigma} = \delta_\mu{}^\rho\delta_\nu{}^\sigma - \delta_\mu{}^\sigma\delta_\nu{}^\rho$ is an identity matrix and $C^n = C_{\mu\nu}{}^{\alpha_1\beta_1}C_{\alpha_1\beta_1}{}^{\alpha_2\beta_2}\ldots C_{\alpha_{n-1}\beta_{n-1}}{}^{\mu\nu}$. In the above equations (2) and (4), we have introduced the factor of L so that the coupling parameters g_F and $\gamma_{i,j}$ are dimensionless. But for later convenience, we shall work in units where $L = 1$ in what follows. Notice that we shall set $g_F = 1$ in the following numerical calculation. When $X_{\mu\nu}{}^{\rho\sigma} = I_{\mu\nu}{}^{\rho\sigma}$, the action S_A reduces to the standard version of Maxwell theory. For convenience, we denote $\gamma_{1,1} = \gamma$ and $\gamma_{2,i} = \gamma_i (i = 1, 2)$. In this paper, we mainly focus on the 6 derivative terms, i.e., γ_1 and γ_2 terms. But since the effect of γ_1 and γ_2 terms is similar, we only turn on γ_1 term through this paper. Note that when other parameters are turned off, γ_1 is constrained in the region $\gamma_1 \leq 1/48$ in SS–AdS black brane background [15]. The upper bound of γ_1 is because of the requirement that the DC conductivity in the boundary theory is positive [15].

The action S_Ψ supports a superconducting black brane [17]. Ψ is the charged complex scalar field with mass m and the charge q of the Maxwell field A, which can be written as $\Psi = \psi e^{i\theta}$ with ψ being a real scalar field and θ a Stückelberg field. $D_\mu = \partial_\mu - iqA_\mu$ is the covariant derivative. For convenience, we choose the gauge $\theta = 0$ and then, the EOMs of gauge field and scalar field can be derived as

$$\nabla_\nu\big(X^{\mu\nu\rho\sigma}F_{\rho\sigma}\big) - 4q^2A^\mu\psi^2 = 0\,, \qquad \big[\nabla^2 - (m^2 + q^2A^2)\big]\psi = 0\,. \tag{5}$$

3. Condensation

In this section, we numerically construct a charged scalar hair black brane solution with 6 derivative term[2] and study its superconducting phase transition. The ansatz for the scalar field and gauge field is taken as

$$\psi = \psi(u)\,, \quad A = \mu(1-u)a(u)dt\,, \tag{6}$$

where μ is the chemical potential of the dual field theory. The background EOMs can be explicitly written as

$$\psi'' + \Big(\frac{f'}{f} - \frac{2}{u}\Big)\psi' - \frac{m^2}{u^2 f}\psi - q^2\frac{(u-1)^2\mu^2}{f^2}a^2\psi = 0\,, \tag{7}$$

$$a'' + \Big(\frac{X_3'}{X_3} + \frac{2}{u-1}\Big)a' + \Big(\frac{X_3'}{(u-1)X_3} - \frac{2q^2\psi^2}{u^2 f X_3}\Big)a = 0\,, \tag{8}$$

where the prime represents the derivative with respect to u and for convenience, the tensor $X_{\mu\nu}{}^{\rho\sigma}$ is denoted as $X_A{}^B = \{X_1(u), X_2(u), X_3(u), X_4(u), X_5(u), X_6(u)\}$ with $A, B \in \{tx, ty, tu, xy, xu, yu\}$. This system is characterized by the dimensionless quantities $\hat{T} \equiv T/\mu$. Without loss of generality, we set $m^2 = -2$ in what follows. It is easy to see that the asymptotical behavior of ψ at the boundary $u = 0$ is

$$\psi = u\psi_1 + u^2\psi_2\,. \tag{9}$$

Here, we treat ψ_1 as the source and ψ_2 as the expectation value. And then, we set $\psi_1 = 0$ such that the condensate is not sourced.

We firstly work out the phase diagram of the critical temperature $\hat{T}_c$ for the formation of the superconducting phase as the function of the coupling parameter γ_1. It is convenient to estimate $\hat{T}_c$ by finding static normalizable mode of charged scalar field on the fixed background. This method has been described detailedly in [40,41]. Our result for the phase diagram $(\gamma_1, \hat{T}_c)$ is showed in the left plot in Fig. 1. The red points in this figure denote $\hat{T}_c$ for the representative γ_1, which is obtained by fully solving the coupled EOMs (7) as well as (8) and listed in Table 1. We see that the $\hat{T}_c$ by finding static normalizable mode is in agreement with that shown in Table 1. From the phase diagram $(\gamma_1, \hat{T}_c)$, we find that with the decrease of γ_1, $\hat{T}_c$ decreases. In particular, for the small γ_1, $\hat{T}_c$ seems to approach a fixed value. Therefore, it seems reasonable to infer that the 6 derivative term doesn't spoil the formation of charged scalar hair even if the absolute value of γ_1 $(\gamma_1 < 0)$ is large enough. But note that the HD terms shall be introduced as a perturbative effect, so we shall restrict γ_1 in a small region, $|\gamma_1| \ll 1$. However, in order to see a more obvious effect from 6 derivative term, we also relax the restriction of γ_1 and study the effect of $\gamma_1 = -1$, in which the normal state has a sharp Drude peak and has been studied in [15].

Now, we solve the coupling EOMs (7) and (8) numerically and study the condensation behavior. The result of the condensation $\sqrt{\langle O_2\rangle}/T_c$ as a function of the temperature T/T_c is shown in the right plot in Fig. 1. We observe that with the decrease of γ_1, the

and the quasi-topological gravity [35,36]. But the value of ω_g/T_c is always greater than 8.

[2] As far as we know, the studies on 4 derivative holographic superconductor in the existing literatures except [31] are focus on the 5 dimensional bulk spacetime. Though the qualitative properties are expected to be similar between the 4 and 5 dimensional holographic superconductor with 4 derivative term, we also present the main properties in Appendix A so that the paper is self-contained.