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Ultra-spinning exotic compact objects supporting static massless scalar field configurations

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ABSTRACT

Horizonless spacetimes describing highly compact exotic objects with reflecting (instead of absorbing) surfaces have recently attracted much attention from physicists and mathematicians as possible quantum-gravity alternatives to canonical classical black-hole spacetimes. Interestingly, it has recently been proved that spinning compact objects with angular momenta in the *sub*-critical regime $\bar{a} \equiv J/M^2 \leq 1$ are characterized by an *infinite* countable set of surface radii, $\{r_{\text{c}}(\bar{a};n)\}_{n=1}^{n=\infty}$, that can support asymptotically flat static configurations made of massless scalar fields. In the present paper we study *analytically* the physical properties of *ultra*-spinning exotic compact objects with dimensionless angular momenta in the complementary regime $\bar{a} > 1$. It is proved that ultra-spinning reflecting compact objects with dimensionless angular momenta in the *super*-critical regime $\sqrt{1-[m/(l+2)]^2} \leq |\bar{a}|^{-1} < 1$ are characterized by a *finite* discrete family of surface radii, $\{r_{\text{c}}(\bar{a};n)\}_{n=1}^{n=N_{\text{r}}}$, distributed symmetrically around $r = M$, that can support spatially regular static configurations of massless scalar fields (here the integers $\{l, m\}$ are the harmonic indices of the supported static scalar field modes). Interestingly, the largest supporting surface radius $r_{\text{c}}^{\max}(\bar{a}) \equiv \max_n\{r_{\text{c}}(\bar{a};n)\}$ marks the onset of superradiant instabilities in the composed ultra-spinning-exotic-compact-object-massless-scalar-field system.

1. Introduction

Curved black-hole spacetimes with absorbing event horizons are one of the most exciting predictions of the classical Einstein field equations. The physical and mathematical properties of classical black-hole spacetimes have been extensively explored during the last five decades [1,2], and it is widely believed that the recent detection of gravitational waves [3,4] provides compelling evidence for the existence of spinning astrophysical black holes of the Kerr family. Intriguingly, however, the physical properties of highly compact *horizonless* objects have recently been explored by many physicists (see [5–22] and references therein) in an attempt to determine whether these exotic curved spacetimes can serve as valid alternatives, possibly within the framework of a unified quantum theory of gravity, to canonical black-hole spacetimes.

In a very interesting work, Maggio, Pani, and Ferrari [17] have recently explored the complex resonance spectrum of massless scalar fields linearly coupled to horizonless spinning exotic compact objects. The numerical results presented in [17] have explicitly demonstrated the important physical fact that, for given values $\{l, m\}$ of the scalar field harmonic indices, there is a critical compactness parameter characterizing the central reflecting objects, above which the massless scalar fields grow exponentially in time. This characteristic behavior of the fields in the horizonless spinning curved spacetimes indicates that the corresponding exotic objects may become unstable when coupled to bosonic (integer-spin) fields [23]. In particular, this superradiant instability [24–28] is attributed to the fact that the characteristic absorbing boundary conditions of classical black-hole spacetimes have been replaced in [17] by reflecting boundary conditions at the compact surfaces of the horizonless exotic objects.

The physical properties of *marginally*-stable spinning exotic compact objects were studied analytically in [19]. In particular, it was explicitly proved in [19] that reflecting compact objects with *sub*-critical angular momenta in the regime $0 < \bar{a} \equiv J/M^2 \leq 1$ [29,30] are characterized by an *infinite* countable set of surface radii, $\{r_{\text{c}}(\bar{a};n)\}_{n=1}^{n=\infty}$, which can support spatially regular static (marginally-stable) configurations made of massless scalar fields. The ability of spinning compact objects to support static scalar field configurations is physically interesting from the point of view

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of the no-hair theorems discussed in [31–33]. In particular, it was proved in [31,32] that spherically-symmetric (non-spinning) horizonless reflecting objects, like black holes with absorbing horizons [34–36], cannot support spatially regular nonlinear massless scalar field configurations [37–39].

Interestingly, the parameter space of the composed spinning-exotic-compact-object-massless-scalar-field system is divided by the outermost supporting radius, $r_c^{\max}(\bar{a}) \equiv \max_n\{r_c(\bar{a};n)\}$, to stable and unstable configurations. In particular, horizonless reflecting objects whose surface radii lie in the regime $r_c > r_c^{\max}(\bar{a})$ are stable to scalar perturbation modes [17,19], whereas the ergoregion of compact enough spinning objects in the physical regime $r_c < r_c^{\max}(\bar{a})$ can trigger superradiant instabilities in the surrounding bosonic clouds [17,19].

The main goal of the present paper is to explore the physical properties of exotic *ultra*-spinning ($\bar{a} > 1$) horizonless compact objects [40–43]. Interestingly, we shall explicitly prove below that spinning compact objects in the *super*-critical $\bar{a} > 1$ regime are characterized by a *finite* discrete family of surface radii, $\{r_c(\bar{a};n)\}_{n=1}^{n=N_r}$ [44], that can support the static (marginally-stable) scalar field configurations. This unique property of the *ultra*-spinning ($\bar{a} > 1$) reflecting compact objects should be contrasted with the previously proved fact [19] that sub-critical ($\bar{a} < 1$) spinning objects are characterized by an *infinite* countable family of surface radii, $\{r_c(\bar{a};n)\}_{n=1}^{n=\infty}$, that can support spatially regular static scalar field configurations.

Using analytical techniques, we shall determine in this paper the characteristic critical (largest) surface radius, $r_c^{\max}(\bar{a}) \equiv \max_n\{r_c(\bar{a};n)\}$, of the ultra-spinning reflecting objects that, for given value of the super-critical rotation parameter $\bar{a}$, marks the boundary between stable and superradiantly unstable spinning configurations. In particular, below we shall derive a remarkably compact analytical formula for the discrete (and finite) family of supporting surface radii which characterizes exotic near-critical spinning horizonless compact objects in the physically interesting regime $0 < \bar{a} - 1 \ll 1$.

2. Description of the system

We consider a spatially regular configuration made of a massless scalar field Ψ which is linearly coupled to an ultra-spinning reflecting compact object of radius r_c, mass M, and dimensionless angular momentum in the super-critical regime

$$\bar{a} \equiv \frac{J}{M^2} > 1 \; . \tag{1}$$

Following the interesting physical model of the exotic compact objects discussed by Maggio, Pani, and Ferrari [17] (see also [18–20]), we shall assume that the external spacetime geometry of the spinning compact object is described by the Kerr line element [1,2,29, 45–50]

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\big[adt - (r^2+a^2)d\phi\big]^2 \quad \text{for} \quad r > r_c \; , \tag{2}$$

where the metric functions are given by $\Delta \equiv r^2 - 2Mr + a^2$ and $\rho^2 \equiv r^2 + a^2\cos^2\theta$ with $a \equiv M\bar{a}$.

The spatial and temporal behavior of the massless scalar field configurations in the curved spacetime (2) of the spinning reflecting object is governed by the compact Klein–Gordon wave equation [51,52]

$$\nabla^\nu\nabla_\nu\Psi = 0 \; . \tag{3}$$

Using the spatial-temporal expression [51–53]

$$\Psi(t,r,\theta,\phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta;a\omega) R_{lm}(r;M,a,\omega) e^{-i\omega t} \tag{4}$$

for the linearized massless scalar field, one finds the ordinary differential equation [51,52]

$$\Delta\frac{d}{dr}\Big(\Delta\frac{dR_{lm}}{dr}\Big) + \Big\{[\omega(r^2+a^2)-ma]^2 + \Delta(2ma\omega - K_{lm})\Big\}R_{lm} = 0 \tag{5}$$

for the radial part $R_{lm}(r;M,a,\omega)$ of the massless scalar eigenfunction. The frequency-dependent eigenvalues $K_{lm}(a\omega)$ of the familiar spheroidal harmonic functions $S_{lm}(\theta;a\omega)$ [51,52,54–58] are given by the small frequency $a\omega \ll 1$ expression

$$K_{lm} - a^2\omega^2 = l(l+1) + \sum_{k=1}^{\infty} c_k (a\omega)^{2k} \; , \tag{6}$$

where the explicit functional expression of the coefficients $\{c_k = c_k(l,m)\}$ is given in [56].

Following the interesting physical models discussed in [17–20] for horizonless curved spacetimes, we shall assume that the scalar fields vanish on the compact reflecting surfaces of the central exotic compact objects [59]:

$$R(r = r_c) = 0 \; . \tag{7}$$

In addition, we consider asymptotically flat linearized scalar field configurations which are characterized by asymptotically decaying radial eigenfunctions:

$$R(r\to\infty) \to 0 \; . \tag{8}$$

3. The resonance condition of the composed ultra-spinning-exotic-compact-object-massless-scalar-field configurations

In the present section we shall derive, for a given set of the dimensionless physical parameters $\{r_c/M, \bar{a}, l, m\}$, the characteristic resonance condition for the existence of ultra-spinning reflecting exotic horizonless objects that support spatially regular *static* (marginally-stable) linearized scalar field configurations.

Substituting into the radial equation (5) the characteristic relation

$$\omega = 0 \tag{9}$$

for the static scalar field configurations, one obtains the ordinary differential equation [19,60]

$$x(1-x)\frac{d^2F}{dx^2} + \{(1-\gamma) - [1 + 2(l+1) - \gamma]x\}\frac{dF}{dx} - [(l+1)^2 - \gamma(l+1)]F = 0 \; , \tag{10}$$

where

$$R(x) = x^{-\gamma/2}(1-x)^{l+1}F(x) \; , \tag{11}$$

$$x \equiv \frac{r - M(1 + i\sqrt{\bar{a}^2-1})}{r - M(1 - i\sqrt{\bar{a}^2-1})} \; , \tag{12}$$

and

$$\gamma \equiv \frac{m}{\sqrt{1-\bar{a}^{-2}}} \; . \tag{13}$$

The physically acceptable solution of the characteristic radial scalar equation (10) which respects the asymptotic boundary condition (8) is given by [19,56,61,62]